

## Function Clarification

This note should clarify the confusion over functions from class on Tuesday. As stated in the book,

A function is a rule that assigns to each element  $x \in X$  exactly one element  $y \in Y$

This definition is not as restrictive as I had originally thought. The requirements for a function is that every element in your domain gets mapped to only one element in your codomain. Thus you need to make sure that everything in your domain has an output value, and that the same point does not map to two different points in your codomain. If you have a graph, you can test to see if it is a function by the vertical line test. This makes sure that nothing in your domain (the  $x$  axis) goes to two different points, which the vertical line would intersect twice.

I confused functions with a 1-1 property, which states that whenever  $f(x_1) = f(x_2)$  in  $Y$ , then  $x_1 = x_2$  in  $X$ . Graphically this is represented by the horizontal line test. If a horizontal line intersects your function in two places, then it is not 1-1. The reason for this test is we want to see if  $f$  is invertible, or if we can assign a function  $g$  going from  $Y$  to  $X$  such that  $g(f(x)) = x$ . If this is the case, then  $g = f^{-1}$ . If  $f$  was not 1-1, then we cannot find  $f^{-1}$  because everything in our codomain  $Y$  does not map to exactly one element in our domain  $X$ . Figure 1 should illustrate this point.

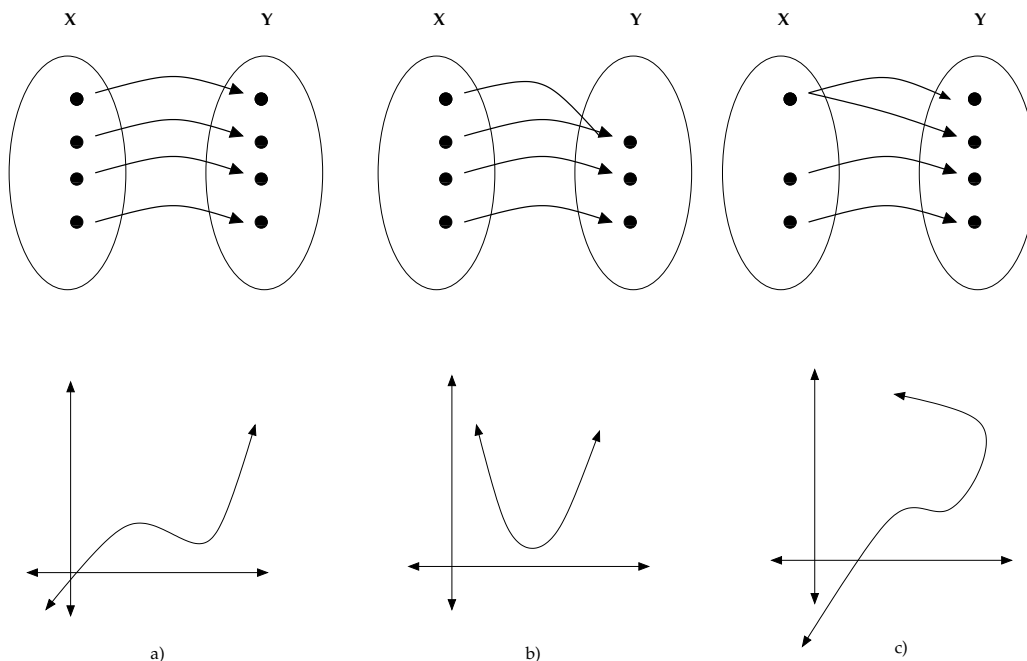


Figure 1: a) This is a graph of an invertible function-every element in the domain  $X$  is mapped to exactly one element in the codomain  $Y$ , and every element in the codomain is associated with exactly one element in the domain. Both the vertical and horizontal line tests succeed. b) This is a function because every element in the domain is mapped to exactly one element in the codomain  $Y$ , but it is not an invertible function because two elements in the domain are mapped to the same element in  $Y$ . Notice in the graph that the vertical line test succeeds, but the horizontal line test fails. c) This is not a function because an element in  $X$  maps to two different elements in the codomain.