Key

Math 1100-5
Fall 2004

Integral Practice

Instructions Integrate the following functions using the techniques and rules developed in class. Recall the rule of thumb: simplify only when needed or for clarity. This does **not** need to be handed in.

1. $\int \sqrt{2}\left(\sqrt{2}x+1\right)^3\ dx$

$$\frac{1}{4}\left(\sqrt{2}x+1\right)^4+C$$

2. $\int_0^3 \left(\pi x^3+1\right)^4\ 3\pi x^2\ dx$

$$\frac{1}{5}\left(\pi x^3+1\right)^5\Big|_{x=0}^{3} = \frac{1}{5}\left(27\pi+1\right)^5-\frac{1}{5}$$

3. $\int \frac{3y}{\sqrt{2y^2+5}}\ dy$

$$\frac{3}{2}\sqrt{2y^2+5}$$

4. $\int_{-1}^3 2x^2-8\ dx$

$$\frac{2}{3}x^3-8x\Big|_{-1}^{8} = \frac{-40}{3}$$

5. $\int_0^4 \sqrt{x^2+x}\ (2x+1)\ dx$

$$= \frac{2}{3}\left(x^2+x\right)^{3/2}\Big|_0^4$$

$$= \frac{2}{3}\,20^{3/2} \simeq 59.63$$

6. $\int_0^1 \frac{x+1}{(x^2+2x+6)^2}\ dy$

$$-\frac{1}{2}\left(x^2+2x+6\right)^{-1}\Big|_{x=0}^{1} = \frac{1}{36}$$

7. $\int xe^{x^2-3}\ dx$

$$\frac{1}{2}e^{(x^2-3)}+C$$

8. $\int \frac{e^{3/x}}{x^2}\ dx$

$$-\frac{1}{3}e^{3/x}+C$$

9. $\int \frac{x^2-2x+4}{x^2-2}\ dx$

$$\int \frac{(x-2)^2}{(x+2)(x-2)}\,dx = \int \frac{x-2}{x+2}\,dx$$

$u = x+2$

$du = dx$

$$\int \frac{x+2-4}{x+2} = \int \frac{u-4}{u}\,du = \frac{1}{8}u^2-4\ln u$$

$$= \frac{1}{8}(x+2)^2-4\ln(x+2)+C$$

#10

producers surplus:

$$P_1 x_1 - \int_0^{x_1} \text{supply Function } dx$$

supply function: $4x+4$

equilibrium price:

$$4x+4 = 49 - x^2$$

$$x^2 + 4x - 45 = 0$$

$$(x+9)(x-5) = 0 \Rightarrow x = -9, 5$$

so $x_1 = 5$, $P_1 = 24$

Producers surplus

$$24(5) - \int_0^5 4x+4 \, dx$$

$$= 120 - \left(2x^2+4x\right)\Big|_{x=0}^{5} = \boxed{\$50}$$

#11. a. Not continuous

b. cannot be integrated from 0 to 4 because the Fundamental Theorem of calculus requires $f(x)$ to be a continuous function.

c. The area under the curve can be found with geometry

$$32 + 64 + 96 + 128 = 320$$

#12 Area between curve $y = 2x, \quad x^3 - 2x$

Find intersection: $x^3 - 2x = 2x \Rightarrow x^3 - 4x = 0$

$x(x^2 - 4) = 0$

$x = 0, \quad x = \pm 2$

Area between curve

$$\int_{-2}^{0} x^3 - 2x - 2x \,)\, dx + \int_{0}^{2} 2x - (x^3 - 2x)\, dx$$

$$\frac{1}{4}x^4 - 2x^2 \Big|_{x=-2}^{0} + 2x^2 - \frac{1}{4}x^4 \Big|_{x=0}^{2} = -(4-8) + (8-4)$$

$$= 8$$

#13 a. optimal level of production: $3x + 20 = 44 - 5x \Rightarrow x = 3$

b. Cost: $\int 3x + 20 \, dx = \frac{3x^2}{2} + 20x + K = C(x)$

$C(80) = 11200 + K = 11400, \quad$ so $K = 200$

Revenue: $\int 44 - 5x \, dx = 44x - \frac{5x^2}{2} + K = R(x)$

$R(0) = 0, \quad$ so $K = 0$

b. Profit $= P(x) = R(x) - C(x) = 24x - 4x^2 - 200$

c. $P(3) = -164$ (a loss)

14. Find equilibrium price

$$\frac{12}{x+1} = 1 + 0.2x \quad \Rightarrow \quad 12 = 1 + 1.2x + 0.2x^2$$

$$2x^2 + 12x - 110 = 0$$

$$2(x+11)(x-5) = 0$$

$x_1 = 5 \quad p_1 = 2$ $\qquad$ $x = 5$

Consumer surplus

$$\int_0^5 \frac{12}{x+1}\,dx - 5 \cdot 2 = 12 \ln(x+1) \Big|_{x=0}^{5} - 10 = \$11.50$$

15. $$\int_{-2}^{2} x^2 + 2 - (-x)\,dx = \int_{-2}^{2} x^2 + x + 2\,dx$$

$$= \frac{1}{3}x^3 + \frac{1}{2}x^2 + 2x \Big|_{x=-2}^{2} = \frac{40}{3}$$