

Math 1100-5  
Fall 2004

Name: \_\_\_\_\_

### Integral Practice

**Instructions** Solve the following problems involving integrals. These problems are drawn from Sections 12.4, 13.2, and 13.3, so refer back to those sections if you are stuck. This worksheet is due on November 16.

1. Find the area enclosed by the functions  $g(x) = 6/x$  and  $r(x) = -x - 5$ .

2. The demand function for a certain product is given by:

$$p = 500 + \frac{1000}{q + 1}$$

where  $p$  is the price and  $q$  is the number of units demanded. Find the average price as demand ranges from 49 to 99 units.

3. **a.** Integrate the following functions:

$$\int_{-2}^2 x^2 dx \quad \text{and} \quad \int_{-2}^2 x dx$$
$$\int_0^2 x^2 dx \quad \text{and} \quad \int_0^2 x dx$$

by calculating the definite integral

**b.** Graph both of those functions, and sketch the area under the curve that you are integrating. Do you notice anything unusual about these functions that helps to explain your answers above?

4. The average cost of a product changes at the rate:

$$\overline{C}'(x) = -6x^{-2} + \frac{1}{6}$$

- a.** Find the average cost function, where the average cost of 6 units is \$10.00.  
**b.** Find the average cost of 12 units.

5. A monopoly has a total cost function  $C = 1000 + 120x + 6x^2$  for its product, which has demand function  $p = 360 - 3x - 2x^2$ . Find the consumer's surplus at the point where the monopoly has maximum profit.

6. Suppose that the supply function for a good is  $p = 0.1x^2 + 3x + 20$ . If the equilibrium price is \$36, what is the producer's surplus there?

### Integral Practice

**Instructions** Integrate the following functions using the techniques and rules developed in class. Recall the rule of thumb: simplify only when needed or for clarity. This does **not** need to be handed in.

1.  $\int \sqrt{2} (\sqrt{2}x + 1)^3 dx$

6.  $\int_0^1 \frac{x+1}{(x^2+2x+6)^2} dy$

2.  $\int_0^3 (\pi x^3 + 1)^4 3\pi x^2 dx$

7.  $\int x e^{x^2-3} dx$

3.  $\int \frac{3y}{\sqrt{2y^2+5}} dy$

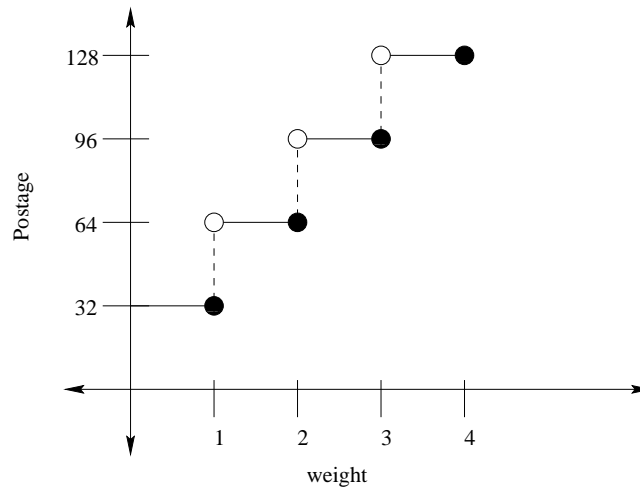
8.  $\int \frac{e^{3/x}}{x^2} dx$

4.  $\int_{-1}^3 2x^2 - 8 dx$

9.  $\int \frac{x^2-2x+4}{x^2-2} dx$

5.  $\int_0^4 \sqrt{x^2+x} (2x+1) dx$

10. Find the producer's surplus for a product if its demand function is  $p = 49 - x^2$  and its supply function is  $p = 4x + 4$ .
11. Consider the following "step function" in Figure 11. (A related problem done in homework is Section 9.2, number 56).



- a. Is this function continuous?
- b. Can this function be integrated from 0 to 4? Why or why not?
- c. Can you find the area under the curve 0 to 4? What is it?
12. Find the area between the curve of  $y = 2x$  and  $y = x^3 - 2x$
13. A firm knows that its marginal cost for a product is  $\overline{MC} = 3x + 20$ , that its marginal revenue  $\overline{MR} = 44 - 5x$ , and that the cost of production and sale of 80 units is \$ 11,400.
- a. Find the optimal level of production.
- b. Find the profit function.
- c. Find the profit or loss at the optimal level.
14. If the demand function for a product is  $p = 12/(x + 1)$  and the supply function is  $p = 1 + 0.2x$ , find the consumer's surplus under pure competition.
15. Find the area between the functions  $f(x) = x^2 + 2$  and  $y(x) = -x$  from  $x = -2$  to  $x = 2$ .