

KEY

Name: _____

Integral Practice

Instructions Solve the following problems involving integrals. These problems are drawn from Sections 12.4, 13.2, and 13.3, so refer back to those sections if you are stuck. This worksheet is due on November 16.

1. Find the area enclosed by the functions $g(x) = 6/x$ and $r(x) = -x - 5$.

$$\frac{6}{x} = -x - 5 \quad \Rightarrow \quad 6 = -x^2 - 5x \quad x^2 + 5x + 6 = 0, \quad (x+3)(x+2) = 0$$

$$x = -2, -3$$

$$\int_{-3}^{-2} \left(\frac{6}{x} - (-x-5) \right) dx = 6 \ln x + \frac{1}{2}x^2 + 5x \Big|_{-3}^{-2} = 6(\ln(-2) + 2 - 10) - \left(6 \ln(-3) + \frac{9}{2} - 15 \right)$$

$$= 6 \ln(-2) - 6 \ln(-3) + \frac{5}{2} \approx .0672$$

2. The demand function for a certain product is given by:

$$p = 500 + \frac{1000}{q+1}$$

where p is the price and q is the number of units demanded. Find the average price as demand ranges from 49 to 99 units.

Average price:

$$\frac{1}{99-49} \int_{49}^{99} \left(500 + \frac{1000}{q+1} \right) dq = \frac{1}{50} \left(500q + 1000 \ln(q+1) \right) \Big|_{q=49}^{99}$$

$$= \frac{1}{50} (1000 \ln 2 + 25000) = \boxed{\$513.86}$$

3. a. Integrate the following functions:

$$\int_{-2}^2 x^2 dx \quad \text{and} \quad \int_{-2}^2 x dx$$

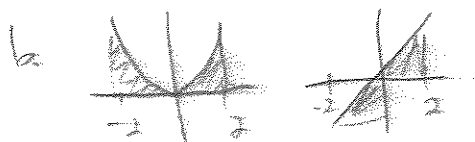
$$\int_0^2 x^2 dx \quad \text{and} \quad \int_0^2 x dx$$

by calculating the definite integral

b. Graph both of those functions, and sketch the area under the curve that you are integrating. Do you notice anything unusual about these functions that helps to explain your answers above?

a. $\int_{-2}^2 x^2 dx = \frac{1}{3}x^3 \Big|_{-2}^2 = \frac{16}{3}$ $\int_{-2}^2 x dx = \frac{1}{2}x^2 \Big|_{-2}^2 = 0$

$\int_0^2 x^2 dx = \frac{8}{3}$ $\int_0^2 x dx = \frac{1}{2}$



x^2 is symmetric about y axis \Rightarrow integral on a symmetric interval $[-2, 2]$ is twice integral from 0 to 2

x has equal area above & below x-axis

4. The average cost of a product changes at the rate:

$$\bar{C}'(x) = -6x^{-2} + \frac{1}{6}$$

- a. Find the average cost function, when the cost of making 6 units is \$10
 b. Find the average cost of 12 units.

a. $\bar{C}(x) = \int -6x^{-2} + \frac{1}{6} dx = 6x^{-1} + \frac{1}{6}x + K$

$\bar{C}(6) = \frac{6}{6} + \frac{1}{6} \cdot 6 + K = 2 + K = 10 \Rightarrow K = 8$

a. $\bar{C}(x) = 6x^{-1} + \frac{1}{6}x + 8$

b. $\bar{C}(12) = \frac{6}{12} + \frac{12}{6} + 8 = \10.50

5. A monopoly has a total cost function $C = 1000 + 120x + 6x^2$ for its product, which has demand function $p = 360 - 3x - 2x^2$. Find the consumer's surplus at the point where the monopoly has maximum profit.

$$\text{Revenue} = px = x(360 - 3x - 2x^2)$$

$$\text{Revenue} = \text{Profit} - \text{Cost}$$

$$\text{so Profit} = \text{Revenue} - \text{Cost} = 360x - 3x^2 - 2x^3 - 1000 - 120x - 6x^2$$

$$P(x) = -2x^3 - 9x^2 + 240x - 1000$$

$$P'(x) = -6x^2 - 18x + 240 \Rightarrow P' = 0 \Rightarrow -6(x^2 + 3x - 40) \\ = -6(x+8)(x-5)$$

Maximum profit at $x=5$ w/ $p(5) = 295$

Consumer Surplus =

$$\int_0^5 (360 - 3x - 2x^2) dx - (295)(5) = \left[360x - \frac{3}{2}x^2 - \frac{2}{3}x^3 \right]_0^5 - 1475 \\ = 1800 - 37.50 - 83.33 - 1475 = \boxed{\$ 204.17}$$

6. Suppose that the supply function for a good is $p = 0.1x^2 + 3x + 20$. If the equilibrium price is \$36, what is the producer's surplus there?

$$36 = 0.1x^2 + 3x + 20$$

$$360 = x^2 + 30x + 200$$

$$x^2 + 30x - 160 = 0$$

$$x = \frac{-30 \pm \sqrt{900 + 640}}{2} = 4.62$$

Producer's Surplus:

$$(36)(4.62) - \int_0^{4.62} (0.1x^2 + 3x + 20) dx$$

$$166.32 - \left(\frac{0.1}{3}x^3 + \frac{3}{2}x^2 + 20x \right) \Big|_{x=0}^{4.62}$$

$$= 166.32 - 127.7 = \boxed{\$ 38.62}$$

