Final Key

1. $\int_{-\infty}^{2} e^{x}\,dx = \lim_{a\to\infty} \int_{-a}^{2} e^{x}\,dx = \lim_{a\to\infty} e^{x}\Big|_{-a}^{2} = \lim_{a\to\infty} e^{2} - e^{-a} \to e^{2}$

2. $f(x,y) = \frac{2x}{x+y}$ domain: all $x \neq y$ except $x=-y$

$$\frac{\partial f}{\partial x} = \frac{2(x+y)-2x}{(x+y)^2} = \frac{2y}{(x+y)^2} \qquad \frac{\partial f}{\partial y} = \frac{-2x}{(x+y)^2} \qquad \frac{\partial f}{\partial y\,\partial x} = \frac{2(x+y)^2 - 4y(x+y)}{(x+y)^4}$$

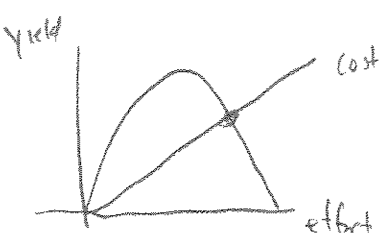
3. $P = A x^{\alpha} y^{\beta}$

a. $\alpha+\beta>1$ does not have constant returns to scale because $f(ax,ay) \neq af(x,y)$

b. $\alpha = 1/4$ $\beta = 3/4$ $A=100$

$$\frac{\partial P}{\partial x} = 25\, x^{-3/4} y^{3/4}$$

4.



Being exploited
($E_{MSY} > E^{*}$)

Group 2

5. $\lim_{x\to 1} f(x) = \begin{cases} x^2-10 & x<1 \\ \sqrt{x}+51 & x\geq 1 \end{cases}$ $\Rightarrow$ $\lim_{x\to 1^-} f = -9$ limit DNE

$\lim_{x\to 1^+} f = 52$

6. $f = \left(\sqrt{2x+1}\right)(x^3+1)$

$f' = \frac{1}{2}(2x+1)^{-1/2} \cdot 2 \cdot (x^3+1) + \left(\sqrt{2x+1}\right)(3x^2)$

7 $\int 5x^4 - \frac{4}{x} + \frac{1}{x^3}\, dx = x^5 - 4\ln x - \frac{1}{2}x^{-2} + C$

8. $\int \frac{x^8}{\sqrt{x^9-1}}\, dx =$ $u = x^9-1$, $du = 9x^8\, dx$ $\Rightarrow \frac{1}{9}\int \frac{1}{\sqrt{u}}\, du = \frac{1}{9} \cdot 2u^{1/2} + C$

$= \frac{2}{9}\sqrt{x^9-1} + C$

9. $s = x^2 + \ln x$

$s'' = 2 - \frac{1}{x^2}$

10 $\int_0^3 e^x\, dx = e^x \Big]_0^3 = e^3 - 1$

11. $\lim_{x\to 5} \frac{x^3-2x^2-15x}{x^2-3x-10} = \lim_{x\to 5} \frac{x\cancel{(x-5)}(x+3)}{\cancel{(x-5)}(x+2)} = \frac{5(8)}{7} = \frac{40}{7}$

Group C.

12. $y+8 = xy^2+y$

$\frac{dy}{dx} = y^2 + x\,2y\,\frac{dy}{dx} + \frac{dy}{dx}$ $\quad\quad$ $\frac{dy}{dx}(1-2xy\cdot 1) = y^2$

$\frac{dy}{dx} = \frac{y^2}{-2xy} = -\frac{y}{2x}$

13 $\int x^2 e^{x^3/3}\,dx$ $\quad$ $u = x^3/3$ $\quad \Rightarrow \quad e^{x^3/3} + c$

14 $\lim_{x \to \infty} \frac{9x^2+4x+1}{x^2+10x+24} \longrightarrow 9$

15. $f = xe^{5x}$

$f' = e^{5x} + 5xe^{5x}$

16 $\int_1^3 3x^{1/2} - x^{-4/3}\,dx = 2x^{3/2} + 3x^{-1/3} \Big|_1^3$ $\quad$ 7.47

$2(3)^{3/2} + 3(3)^{-1/3} - (2+3)$

17 $y = \ln\left(\frac{x^2+5}{9x+1}\right)^3 = 3\ln(x^2+5) - 3\ln 9x+1$

$y' = \frac{3}{x^2+5}\cdot(2x) - \frac{3}{9x+1}\cdot 9 = \frac{6x}{x^2+5} - \frac{27}{9x+1}$

18. $\int_0^1 \frac{x}{x^2+5}\,dx$ $\quad$ $u = x^2+5$, $du = 2x\,dx$ $\quad$ $\frac{1}{2}\int \frac{1}{u}\,du = \frac{1}{2}\ln(x^2+5)\Big|_0^1$

$= \frac{1}{2}(\ln 6 - \ln 5)$

Group D

19. $f(x) = 3x^2$ $\rightarrow$ $\lim_{h\to 0} \frac{3(x+h)^2 - 3x^2}{h}$ $\lim_{h\to 0} \frac{3(x^2+2xh+h^2) - 3x^2}{h}$

$f' = 6x$

$= \lim_{h\to 0} \frac{6xh+3h^2}{h} = \lim_{h\to 0} 6x + 3h \rightarrow 6x$

20. $f = x^2 + 3x + x^{3/2}$

$f(25) = 625 + 75 + 125 = 825$

$f' = 2x + 3 + \frac{3}{2}x^{1/2}$

$f'(25) = 2(25) + 3 + \frac{3}{2}(5) = 53 + 7.5 = 60.5$

60.5
$\underline{25}$
$0\ 25$

$y - 825 = 60.5(x - 25)$ $\quad y = 60.5x - 687.5$

21. $\frac{dy}{dx} = 3yx + 15x^2y = 3y(x + 5x^2)$ $\quad \frac{dy}{3y} = x + 5x^2\,dr$

$\frac{1}{3}\ln y = \frac{1}{2}x^2 + \frac{5}{3}x^3 + c$

$\boxed{\frac{1}{3}\ln y = \frac{1}{2}x^2 + \frac{5}{3}x^3 - 49.5}$

$\frac{1}{3}\cancel{\ln 1} = \frac{1}{2}9 + \frac{5}{3}27 + c$

$4.5 + 45$

$c = -49.5$

22. Producers surplus is the added value for the purchase of a product. The consumer would pay more, but it is sold at a lower price

$p = 49 - x^2 = 4x + 4$

$x^2 + 4x - 45 = 0$

$(x+9)(x-5) = 0$

$x_1 = 5$

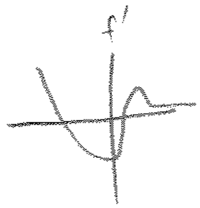
and $p_1 = 24$

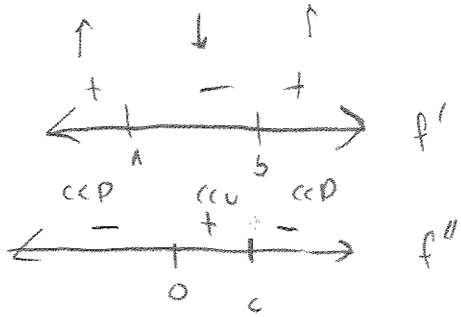
$\int_0^5 49 - x^2 - 24(5)$

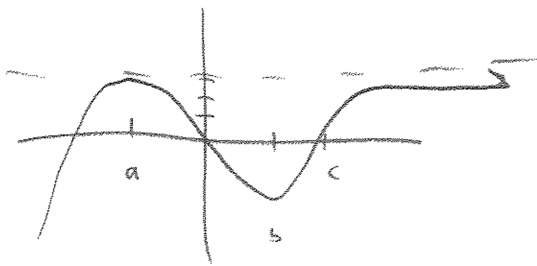
$49x - \frac{1}{3}x^3 \Big|_0^5 - 24(5) = \frac{250}{3} = 83.3$

$\frac{610}{3} - 120$

23.



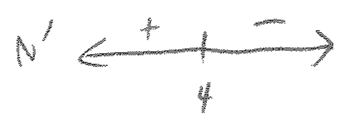




24. $N = 8\left(1 + \dfrac{t}{t^2+16}\right)$

$$N' = 8 \cdot \left(\frac{(t^2+16) - 2t^2}{(t^2+16)^2}\right) = 8 \cdot \left(\frac{16 - t^2}{(t^2+16)^2}\right)$$

$N' = 0 \Rightarrow t = \pm 4$, so



max number of employees is 4 (t = 4 is a maximum)

$$N(4) = 8\left(1 + \frac{4}{16+16}\right) = 8\left(1 + \frac{1}{8}\right) = 9 \text{ employess}$$