

Name: _____

Name of person on your left: _____

Exam II

Instructions: This exam is a total of 90 points. Make sure you have all 5 pages. Answer each question carefully and thoughtfully to receive full credit. Partial credit will be awarded, and points will be deducted if you write the answer down to a problem without justifying your steps. You do not need to simplify your answer unless it helps for clarity.

True/False (4 points each) Answer each question by marking “T” if the statement is true or “F” if the statement is false. If the statement is false, then in the line below write the statement that would make it true OR provide a counterexample.

___ 1. A vertical asymptote for the rational function $f(x)$ may occur when the denominator is 0 for some finite value x .

___ 2. Critical points occur when $f''(x) = 0$ or is undefined.

___ 3. The second derivative of $f(x) = x$ is $1/x$.

___ 4. The maximum average cost $\overline{C(x)}$ occurs where the maximum average cost equals the marginal cost.

___ 5. If $f(x)$ has a local maximum at $x = a$, then the second derivative $f''(a) < 0$.

(10 points) 6. Find any and all vertical and horizontal asymptotes of the following function. Be explicit in your work and reasoning.

$$f(x) = \frac{9x^2 + 4x + 1}{x^2 + 10x + 24}$$

Derivatives (5 points each): Evaluate the following derivatives. Be sure to note what derivative you are computing.

$$7. s(x) = \frac{x+1}{3x+2}$$
$$s''(x) =$$

$$8. y(x) = \ln\left(\frac{x^2+5}{\sqrt{9x+5}}\right)^2$$
$$y'(x) =$$

$$9. f(x) = e^{9x^2} \ln(3e^{2x} + 1)$$
$$f'(x) =$$

$$10. y + 8 = x^2 + 3x^2y^4$$
$$\frac{dy}{dx} =$$

(10 points) 11. Find the equation for the tangent lines of

$$\frac{x^2}{9} + \frac{y^2}{2} - \frac{y}{6} = \frac{7}{18}$$

at $x = 1$. *Hint:* You should have two equations for the tangent line.

(10 points) 12. Suppose that a new company begins production with eight employees and the growth of the company over the next few years is predicted by:

$$N = N(t) = 10 \left(1 + \frac{5t}{t^2 + 9} \right) \text{ for } 0 \leq t \leq 10$$

In what year will the number of employees be maximized and what will be the maximum number of employees?

(10 points): 13. You are given the graph of the first derivative of $f(x)$. Furthermore, you know that $f(x)$ has *no* vertical asymptotes and that:

$$\lim_{x \rightarrow \infty} f(x) = 3 \text{ and } \lim_{x \rightarrow -\infty} f(x) = -\infty$$

Using this information, provide a graph of the function $f(x)$ in Figure 1. Be sure to also consider inflection points and concavity.

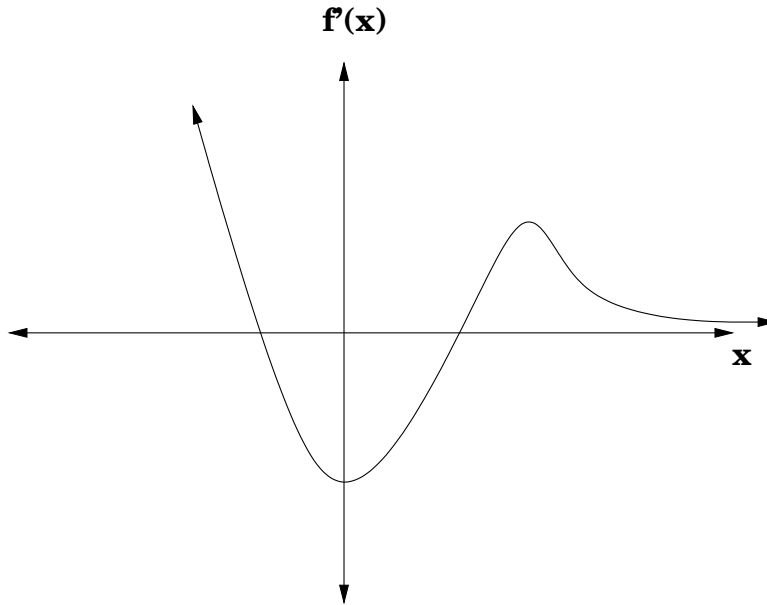


Figure 1: Graph of $f'(x)$

(10 points) 14. Suppose that for a particular product, the number of units x produced per month depends on the number of thousands of dollars y invested, with $x(t) = 90 y(t) + 10 [y(t)]^2$. At what rate will production increase if \$5000 is invested and the investment capital is increasing at a rate of \$750 per month?