

Applications of the Derivative

Instructions Answer all questions on a separate piece of paper. This homework is due September 28.

The CFO of the Navistar corporation, May K. Lott-Amonie, has hired you to determine what marketing strategy she should adopt for the new CXT pickup. In particular, would you recommend that she pursue a strategy that maximizes revenue or maximizes profit? She does recall taking a class at her university a long time ago that used the first edition of *Mathematical Applications: for the Management, Life, and Social Sciences* by Harshbarger and Reynolds. She hazily recalls terms like marginal profit, revenue, cost, but has forgotten how all of them relate.

Part I. Ms. May does know that the cost to manufacture the trucks is given by:

$$C(x) = 30x + 5,$$

where x is the number of trucks made in the thousands and $C(x)$ is given in the thousands of dollars.

1. Since Navistar is the only company on the market with this particular type of truck, at the beginning the revenue will grow linearly. Assume that the price of a new truck will be \$90,000. What is the expression of the revenue function $R(x)$?

2. Can you provide May with the graph of the profit as a function of the number of trucks sold? (Make sure you identify the regions where there is a loss and a profit.)

3. What is the marginal profit in this case?

Part II. Ms. Lott-Amonie has received information that her competitors are developing similar products, and thus the market will become a competitive one. She is asked to develop a marketing strategy for when this occurs. Ms. Lott-Amonie does know that the demand function for her product is given by:

$$p(x) = 100 - x,$$

where $p(x)$ is given in the thousands of dollars.

4. Can you determine the number of trucks x that need to be sold so that the *revenue* is maximized? (Ms. Lott-Amonie asks you to use calculus so she can start to remember what she learned.)

5. Can you also determine the number of trucks x that need to be sold so that the *profit* is maximized?

6. Based on the previous two answers, the CFO decides to pursue a marketing strategy that would maximize the revenue instead of the profit. How can you show her that this would be a bad idea and result in her getting fired?

Part III. The CFO is encouraged by these results you gave her. Surveys conducted of potential consumers show that there is a substantial amount of “buzz” and interest in the CXT, although many consumers were less likely to buy one when they discovered it has poor gas mileage and runs on diesel. This “buzz factor” needs to be taken into consideration when introducing this product to the market. As more products are sold, the buzz is expected to drop. In this situation, one can determine a functional form for “buzz”:

$$B(x) = a - bx^2$$

where a and b are constants bigger than zero. As a result your profit then becomes:

$$P(x) = R(x) - C(x) + B(x)$$

7. Find the x intercept for the “buzz” function and explain how positive and negative buzz will affect the profit. What does the x intercept represent in relation to “buzz” affecting the profit? Using your expression for the x intercept, explain why a big value for a is a good thing for profit, but a big value for b is a bad thing. Last, in your own words, describe what the constants a and b could represent.

8. The financial backers are worried that bad buzz will affect profits, and thus want an initial production level that will still yield a profit. Ms. Lott-Amonie has determined that $a = 20$ and $b = 1$ for the buzz function. Can you find a range of production levels where there will still be a profit? How does this range compare to the production levels that yield a profit when you don’t consider buzz?

9. Finally, provide the CFO with a target sales goal that would maximize the profit.