

Mathematical Bioeconomics

We live in a global economy-in the middle of winter we eat kiwi fruit from New Zealand. Oil that drives our cars comes from the Middle East. McDonalds can be found worldwide. Globalization has impacted our day to day lives for better or for worse. One consequence of globalization is that our individual decisions and actions have an impact on world markets. A second consequence is that small localized events affect consumers price. A deep frost to the orange groves in Florida can radically drive up prices. The importation of a cow with Mad Cow disease was enough to drive up beef prices all over the country.

World population is growing at an ever increasing rate, creating more demand for products and goods. Some of these goods come from *renewable resources*. Agricultural products are a renewable resource because every season a new crop is produced. Fisheries are a renewable resource because fish reproduce. Forests are a renewable resource that can be replanted to grow more wood. Ski resorts are a renewable resource because every year winter snowfall allows us to hit the slopes.

While these resources may be renewable, the quality of the resource may certainly change and lead to the decline and the utility of the resource. Overfishing will lead to a decline in fisheries because it is fished faster than the fish have the opportunity to reproduce. Increasing temperatures may shorten the length of the winter season and the amount of snowfall.

How can we effectively manage these resources to prevent a catastrophic bust to our global economy? Mathematical bioeconomics is the study of the management of renewable resources. It takes into consideration not only economic questions (revenue, cost, etc) but also the impact of this demand on the resource.

One of the guiding principles in mathematical bioeconomics is the “tragedy of the commons.” Say you share a refrigerator with your co-workers. It is in your interest to use it because it keeps your food cold, and you can store items to make a more nutritious lunch. Space is limiting in this refrigerator, but everyone has the same desire as you (store your lunch). Eventually someone will leave the gallon of milk in there that becomes an interesting chemistry experiment. If no one claims it as his responsibility, who should clean the refrigerator because it is common space?

The tragedy of the commons demonstrates that when individual interests intersect in a common arena, individual interests will trump all in the absence of external controls (such as a sign saying that every Friday all workers must cooperate in the cleaning of the fridge). How can we use mathematical bioeconomics to prevent such a tragedy of the commons?

1 Understanding Resource Dynamics

One of the mathematical tools used in bioeconomics is differential equations. Differential equations are used to model the processes affecting a resource, as it is conceptually less challenging to quantify how a population is changing through time. The advantage to this approach is that it is like balancing a checkbook.

The prototypical example used in this worksheet is fisheries, but with appropriate thought similar principles apply to other examples. Fish reproduce at a given rate that is dependent on the number of fish present. If there are more fish around, they will reproduce faster. As a result, if we were only to consider birth processes, the differential equation describing the fish population is the following:

$$\frac{dx}{dt} = rx, \tag{1}$$

where x is the number of fish, and r represents the reproduction rate taken to be a constant. If we have an initial condition $x(0) = x_0$ (some initial population, can you find the particular solution to this differential equation?

When is the population stationary, or not changing? We find the answer to this by setting $dx/dt = f(x)$ equal to 0 and solving for x . Note that this now becomes an algebra (rather than a calculus) problem because we are finding where $f(x) = 0$. Points x^* that accomplish this are *equilibrium points*. For our simple fishery model in Equation 1, $dx/dt = 0$ only when $x = 0$. Thus the population will not change when there are no fish around. This may seem trivially true—if no fish are present, then of course it will stay that way. The benefit of this example is that it formalizes our intuition or our practical notions: if nothing is there, it is not going to change. However not all fisheries can be described in this manner, so more realism is needed!

One way to add realism is to consider the effects of crowding. Space and resources are limiting, and if there are more fish around, there is less space for fish to swim or breed. This may have a negative impact on the fish population and lead to a decline in fish population. Such an effect is called *negative density dependence*. We can model this by the following:

$$\frac{dx}{dt} = \underbrace{rx}_{\text{Birth}} - \underbrace{\frac{rx^2}{K}}_{\text{Crowding}}, \quad (2)$$

where K is called the *carrying capacity*. Notice that the term rx^2/K is nonlinear. Equilibrium points are found where:

$$\frac{dx}{dt} = rx - \frac{rx^2}{K} = rx \left(1 - \frac{x}{K}\right) = 0, \quad (3)$$

which occur when $x = 0$ or when $x = K$. Thus we have the addition of two equilibrium points: one where $x = 0$ the existence of a non-zero equilibrium point $x = K$.

When you have the existence of two or more equilibrium points, you must determine which way the population will head to in the long term. Like we did when graphing functions, we test values in between our equilibrium points to see what equilibrium point our system is heading towards.

2 Incorporating Economics

We have talked about the “bio” in bioeconomics, so now it is time to talk about the “economics.”

This semester we looked at revenue, and how profit is the balance between revenue and costs. For resources a similar notion applies except that no one person really “owns” the resource. Thus the notion of profit is termed “sustainable economic rent,” and we need to determine what our revenue and our cost is.

The total revenue is determined by two things: the market price of the resource and the total number of resource produced (or the yield):

$$\text{Total Revenue} = \text{Price} \times \text{Yield}$$

The yield of the resource will involve two things: the effort spent in harvesting the resource and the abundance of resource. As a result, we can say:

$$\text{Yield} = qEx,$$

where q is termed the *catchability coefficient*. This would represent environmental factors that limit (or enhance) our ability to harvest the resource at the best of our abilities. For example is a fishery is harvested using location equipment for the fish, $q > 1$, but if a fishery is harvested using luck, $q < 1$.

The term E represents the *effort* exerted in harvesting the resource. This could be measured in the number of boats at the fishery, number of hours fishing, etc. The term x still represents the abundance of the resource.

So if market price for the resource is p , then we have:

$$\text{Total Revenue} = \text{Price} \times \text{Yield} = pqEx$$

The total cost of harvesting the resource is proportional to the effort exerted:

$$\text{Total Cost} = cE,$$

where c is a constant that represents external controls on cost (such as the price of gasoline to power motors).

As a result, the profit of harvesting the resource (or the total economic rent) is given by:

$$\text{Economic Rent} = pqEx - cE$$

Harvesting the resource will reduce its abundance, so we need to factor in the yield to our resource dynamics:

$$\frac{dx}{dt} = \underbrace{rx}_{\text{Birth}} - \underbrace{\frac{rx^2}{K}}_{\text{Crowding}} - \underbrace{qEx}_{\text{Harvesting}} \quad (4)$$

3 Putting it all together

Bioeconomics tracks *both* the abundance of the resource as well as the profit made from the resource. So thus we have a system of equations:

$$\frac{dx}{dt} = rx - \frac{rx^2}{K} - qEx, \quad (5)$$

$$\text{Economic Rent} = pqEx - cE \quad (6)$$

To analyze the effect of harvesting the resource, two rules must be followed:

Rule 1: The resource will go to one of its equilibrium values where $dx/dt = 0$.

Rule 2: In an open-access resource, effort (E) will tend to a level E^* where the Economic Rent is 0

Rule 1 is a general mathematical principle. If we were to solve that differential equation to get a solution $x(t)$, in the limit as $t \rightarrow \infty$, x will go towards an equilibrium point. Rule 2 is a consequence of the resource being available to all. The Economic Rent is the difference between Total Revenue and Total Cost. If costs exceed revenue, then people will leave the resource because their efforts are not successful. Similarly, if the revenue is greater than the cost, the effort expended harvesting the resource will increase.

The yield of the resource is found by considering Rule 1 and solving $dx/dt = 0$ for x and subsequently putting that expression into the yield:

$$\text{Yield: } Y(E) = qEx(E) = qE \left(\frac{K(r - qE)}{r} \right) = \frac{qK}{r} (rE - qE^2)$$

Note that this function is quadratic in E . The *maximum sustainable yield* is found by maximizing $Y(E)$. This occurs at values of E where:

$$E_{MSY} = \frac{r}{2q}$$

Now let's consider Rule 1 and Rule 2 together. Let's set both the economic rent and dx/dt equal to zero and solve this system of equations. Note that this is an algebraic problem—not a calculus problem. When we do this, we have:

$$(x^*, E^*) = \begin{cases} \left(\frac{c}{pq}, \frac{r}{q} \left(1 - \frac{c}{pqK} \right) \right) \\ (0, 0) \\ (K, 0) \end{cases}$$

Notice how we have three different possibilities, two of which arise from when $E = 0$. This should make sense: at this level, the resource dynamics do not even consider harvesting. The effort level E^* is called a *bionomic equilibrium* because it has both biological and economic parameters.

It is a fact that any effort expended above the maximum sustainable yield will lead to depletion of the resource. Thus we are comparing E^* and E_{MSY} .

If we are over-exploiting the resource, we can impose controls that will reduce our equilibrium effort to levels at or below E_{MSY} . This is done by changing our *parameter values*, which are r , K , q , c , and p . The first two parameters are biological parameters, whereas q , c , and p are economic parameters.

One way to manage our resource is to increase c , the external cost of harvesting the resource. This could represent imposing user fees to harvest the resource, etc. Imposing controls to reduce E^* is proper management of the resource. If this is not done, then resource depletion will occur, and it will no longer become renewable. By doing this analysis one can investigate the effect that different controls have on management.

4 Suggested Problems

- Given the equation

$$\frac{dx}{dt} = rx - \frac{rx^2}{K},$$

verify the equilibrium points are $x = 0$ and $x = K$. Set $K = 10$, $r = 0.1$. Is the population x increasing or decreasing between $0 \leq x \leq K$?

- Verify that an equilibrium point of the differential equation

$$\frac{dx}{dt} = rx - \frac{rx^2}{K} - qEx$$

is $x(E) = \frac{K(r-qE)}{r}$, and use this expression to find the yield.

- For the following problems, I want you to solve them yourself, not just plug into the formulas we found. You can use the formulas to verify your answer.

Say we have a managed reservoir in Utah that is home to the great sport fish *Bioeconomicus excellentus*, or by its common name “calciscool.” Let $r = 0.1$, $q = 0.5$, $c = 0.2$, $K = 10$, $p = 2$.

- Solve the system of equations:

$$0 = rx - \frac{rx^2}{K} - qEx, \tag{7}$$

$$0 = pqEx - cE \tag{8}$$

to find x^* and E^* .

- Use the same values from the previous problem and graph the yield function $Y(E)$. What value E will maximize the yield?
- Is the resource in danger of being depleted? If so, what management practices would you implement in order to prevent it from depletion?
- A proposed idea to reduce the danger of exploitation of the resource is to increase the carrying capacity K . This could be done by increasing the size of the reservoir. Would this reduce the equilibrium effort E^* so the resource isn't exploited? Why or why not?

5 Resources

1. Clark, Colin W. *Mathematical Bioeconomics: The Optimal Management of Renewable Resources*. 1990. John Wiley & Sons.
2. Pojman, Louis P. *Global Environmental Ethics*. 2000. Mayfield Publishing.