Project No. 4, Math 5765, Due May 1 at 5 pm

1. This problem concerns with a PCA analysis for the stocks in S&P 500.
   (a) Perform a web search for the 10 largest holdings in the S&P 500 index, list the company names, their capitalization, and the weights in the index.
   (b) Using daily close information from the past year, construct the covariance matrix.
   (c) Perform a principal component analysis and find the first 5 eigenvalues and eigenvectors. Compute the percentage of variance that is explained by those 5 components.
   (d) Set up two portfolios based on the first two eigenvectors, back test their performance from the beginning of the year, and compare with the actual index performance and the ETF SPY by plotting the price paths.

2. This is a simplified version of the Avellaneda-Lee statistical arbitrage model:

$$\frac{dS(t)}{S(t)} = \alpha_i + \beta \frac{dI(t)}{I(t)} + dX(t)$$

   where $S(t)$ is the price of the stock at time t, $I(t)$ is the index (ETF) price at t, and $dX(t)$ is the idiosyncratic fluctuation. We assume

$$dX(t) = \kappa(m - X(t))\, dt + \sigma dW(t)$$

   It can be shown that $\mathbb{E}[X(t)] = m,\ \mathrm{Var}[X(t)] = \frac{\sigma^2}{2\kappa}$.
   (a) Pick one of the portfolios of your choice, denote the price by $S(t)$, and let $I(t)$ to be the price of ETF SPY, use the past year data to estimate the model parameters $\alpha, \beta,\ \kappa,\ m$, and σ.
   (b) The following dimensional variable

$$s = \frac{(X(t) - m)\sqrt{2\kappa}}{\sigma}$$

   is introduced to be viewed as a score that indicates the relative price being high or low. Explain the intuition behind this choice.
   (c) When the price difference between your portfolio and SPY is too low, we can enter a position to buy your portfolio and sell the ETF. To be more specifically, we buy one dollar of your portfolio and sell β dollars of the ETF. Explain why this is a meaningful thing to do.
   (d) Build a simple Monte Carlo simulation to test the strategy using the following trading rule:

$$\text{buy } S, \text{ sell } I \text{ if } s < -1.25$$
$$\text{sell } S, \text{ buy } I \text{ if } s > +1.25$$

   Run 100 realizations to see if this strategy is profitable.