Suppose you are given $5,000 to invest in the stock market. Choose three of your favorite stocks and download their daily closing prices from yesterday going back one year. There are many online sources where you can find such information, such as finance.yahoo.com and nasdaq.com. For each stock, the daily returns can be calculated using

$$R_n = \frac{S_n - S_{n-1}}{S_{n-1}},$$

where $S_n$ is the closing price on day $n$. Given a return sample $R_n, n = 1, \ldots, N$ ($N = 252$), we can compute the sample mean, which is an estimate for $\mu \Delta t$ (here $\Delta t = 1/252$ is for one day time period annualized), and sample variance, which is an estimate for $\sigma^2 \Delta t$. Also we can estimate the covariance matrix for these three stocks using one of the standard functions (covar for Excel, and cov for Matlab).

(a) Summarize your findings in an expected return vector and a covariance matrix.

(b) Construct portfolio A by equally allocating among these three stocks.

(c) Use the formula for $w$ from the week 10 notes to construct portfolio B that is the global minimum variance portfolio.

(d) Use the formula for $w$ from the week 10 notes to construct portfolio C that is a minimum variance portfolio with specified $\mu_P = 20\%$.

(e) For each of the portfolios constructed, use the stock price data to simulate the portfolio performance over the last three month period (this procedure is called a backtest simulation), and calculate the portfolio return and estimate the portfolio risk level.

(f) Compare the return and volatility with those of 3 stocks to demonstrate the advantages of your optimal portfolios.

(g) Which portfolio would you recommend to an investor? Briefly discuss your answer.