Project No. 1, Math 5765, Due Feb 10 at 5 pm

1. (30 points) The current treasury market can be summarized as follows, assuming that all rates/yields are quoted as annualized continuous compounding, and all zero-coupon bonds have face value \$1:

   - overnight federal fund rate 0.75%,
   - 6-month bill yield 0.67%,
   - 2-year zero-coupon bond price \$0.976;
   - 5-year zero-coupon bond price \$0.908;
   - 10-year zero-coupon bond price \$0.78;
   - 30-year zero-coupon bond price \$0.395.

   The zero-coupon bond price $Z(0,T)$ and the yield $y(0,T)$ are related via

   $$Z(0,T) = e^{-y(0,T)\cdot T}$$

   Here 0 in the first argument refers to the time-0 price or the time-0 yield. The function $y(0,T)$ is called the current **yield curve** and it is the starting point for any quantitative work in the interest rate market.

   (a). Use the market information provided above and a piecewise linear interpolation to build the current yield curve $y(0,T)$ for $0 \le T \le 30$.

   (b). Based on this function $y(0,T)$, derive the zero-coupon bond price $Z(0,T)$. Compare this function with a direct piecewise linear interpolation of the bond prices, and explain why the interpolation based on y is to be preferred.

   (c). The current continuously compounding forward rate for the period (T_1, T_2) is defined as

   $$f(0,T_1,T_2) = \frac{1}{T_2 - T_1}\left(\log Z(0,T_1) - \log Z(0,T_2)\right)$$

   Evaluate the zero-coupon bond price $Z(0,T)$ generated above at $T_j = 0.25j, j = 0,1,2,\ldots,120$, and calculate a set of 3-month forward rates $f_j, j = 0,1,\ldots,119$. Plot the forward rates $f_j = f(0,T_j,T_{j+1})$ and compare them with the yields $y(0,T_j), j = 0,1,\ldots,119$.

2. (10 points) Suppose you observe on the market, in addition to those prices listed in Problem 1, that there is a 7-year zero-coupon bond with price $Z(0,7) = 0.845$. Is there any arbitrage opportunity? Suggest a bond portfolio to your investor if there is and explain to him/her how it is going to work.

3. (10 points) In this problem, we will use the yield curve obtained in Problem 1 to price a coupon bond and estimate possible price changes.

   (a). Price a coupon bond with an annualized coupon rate 3%, paid semiannually, that matures in exactly 15 years;

   (b). In the event that the Fed Reserve moves to raise the overnight rate by 10 basis points (each basis point is equal to 0.01%), and assume that this would cause the yield curve to have a parallel shift, what would be the change in the bond price? What would be the change in the bond yield?