1. (30 points) The current treasury market can be summarized as follows, assuming that all rates/yields are quoted as annualized continuous compounding, and all zero-coupon bonds have face value $1:

- overnight federal fund rate 0.75%,
- 6-month bill yield 0.67%,
- 2-year zero-coupon bond price $0.976;
- 5-year zero-coupon bond price $0.908;
- 10-year zero-coupon bond price $0.78;
- 30-year zero-coupon bond price $0.395.

The zero-coupon bond price $Z(0, T)$ and the yield $y(0, T)$ are related via

$$Z(0, T) = e^{-y(0, T) \cdot T}$$

Here 0 in the first argument refers to the time-0 price or the time-0 yield. The function $y(0, T)$ is called the current yield curve and it is the starting point for any quantitative work in the interest rate market.

(a). Use the market information provided above and a piecewise linear interpolation to build the current yield curve $y(0, T)$ for $0 \leq T \leq 30$.

(b). Based on this function $y(0, T)$, derive the zero-coupon bond price $Z(0, T)$. Compare this function with a direct piecewise linear interpolation of the bond prices, and explain why the interpolation based on $y$ is to be preferred.

(c). The current continuously compounding forward rate for the period $(T_1, T_2)$ is defined as

$$f(0, T_1, T_2) = \frac{1}{T_2 - T_1} (\log Z(0, T_1) - \log Z(0, T_2))$$

Evaluate the zero-coupon bond price $Z(0, T)$ generated above at $T_j = 0.25j, j = 0, 1, 2, \ldots, 120$, and calculate a set of 3-month forward rates $f_j, j = 0, 1, \ldots, 119$. Plot the forward rates $f_j = f(0, T_j, T_{j+1})$ and compare them with the yields $y(0, T_j), j = 0, 1, \ldots, 119$.

2. (10 points) Suppose you observe on the market, in addition to those prices listed in Problem 1, that there is a 7-year zero-coupon bond with price $Z(0, 7) = 0.845$. Is there any arbitrage opportunity? Suggest a bond portfolio to your investor if there is and explain to him/her how it is going to work.
3. (10 points) In this problem, we will use the yield curve obtained in Problem 1 to price a coupon bond and estimate possible price changes.

(a). Price a coupon bond with an annualized coupon rate 3%, paid semiannually, that matures in exactly 15 years;

(b). In the event that the Fed Reserve moves to raise the overnight rate by 10 basis points (each basis point is equal to 0.01%), and assume that this would cause the yield curve to have a parallel shift, what would be the change in the bond price? What would be the change in the bond yield?