Homework Assignment No. 7, due Monday, 3/27 at 5 pm

1. In the cap-and-trade model we discussed in class, the optimization problem is to find the initial number of permits X_0 such that

$$F(X_0) = P_0 \cdot X_0 + (1+r)^{-1}\mathbb{E}\left[g(X_0)^+ \cdot (P_1 + P)\right]$$

is minimum. Here

$$g(X_0) = Q_0 + Q_1 - X_0 - N$$

is the permit shortage (excess for negative values) at the end of the one-year period. The parameters are given as follows.

- $N = 50$ (tons) is the permit endowment;
- $P_0 = \$20$ is the permit price at $t = 0$, $P_1 = P_0 u$ if the price goes up and $P_0 d$ if the price goes down at $t = 1$;
- $Q_0 = 40$ (tons) is the emission level at $t = 0$, $Q_1 = Q_0 u$ if the emission level goes up and $Q_0 d$ if the emission level goes down at $t = 1$;
- $P = \$100$ is the penalty for each permit shortage unit;
- $r = 6\%$ is the interest rate;
- $u = 1.2$ is the up factor, $d = 1/u$ is the down factor.

There will be four scenarios for P_1 and Q_1. We consider two sets of probabilities as shown in the following tables. Obtain the optimal X_0 by plotting the function $F(X_0)$ and locating the minimum for each case, and explain the difference in the results.

Table 1: Case 1

	$P_1 = P_0 u$	$P_1 = P_0 d$
$Q_1 = Q_0 u$	0.4	0.1
$Q_1 = Q_0 d$	0.4	0.1

Table 2: Case 2

	$P_1 = P_0 u$	$P_1 = P_0 d$
$Q_1 = Q_0 u$	0.45	0.05
$Q_1 = Q_0 d$	0.35	0.15