Homework Assignment No. 6, due Wednesday, 3/22 at 5 pm

1. The Vasicek model for the short interest rate $r(t)$ is

$$dr(t) = a(b - r(t))\, dt + \sigma\, dW_t,$$

with constant parameters a, b and σ. The zero-coupon bond price $P(t, r; T)$ is suggested to have the form

$$P(t, r; T) = A(t, T)e^{-B(t,T)r}$$

for some functions A and B that depend only on t and T. Since $P(t, r)$ satisfies the PDE

$$\frac{\partial P}{\partial t} + a(b - r)\frac{\partial P}{\partial r} + \frac{1}{2}\sigma^2\frac{\partial^2 P}{\partial r^2} = rP, \quad \text{with } P(T, r; T) = 1,$$

it is possible to solve for A and B by reducing the PDE to two ODEs for A and B with respective conditions at $t = T$:

$$A(T, T) = 1, \quad B(T, T) = 0.$$

Derive these ODEs and solve them to obtain solutions for A and B.

2. An interest rate term structure model requires daily calibration - a process to choose the parameters so that the model prices for certain securities match the current market prices. In this exercise, we attempt to calibrate the Vasicek model, that is to choose parameters a, b and σ to fit the market, by comparing the function $P(0, r_0, T)$ obtained in Problem 1 with the market data yield curve you generated in the first project. There are two approaches to fit the data:

   (a) You can choose three points from the yield curve corresponding to securities that you believe are more important to fit, and make sure that the prices from the model match exactly with the market data.

   (b) You can introduce a least square fit so that the total error based on all the securities listed is minimized.

   Proceed with both approaches, generate two sets of parameters a, b, and σ, compare and commit on the results. Notice that negative values are not allowed for a, b, and σ.