# Homework Assignment No. 4, due Monday, 2/13 at 5 pm

1. Let the stock price process be a geometric Brownian motion

$$\frac{dS}{S} = r\,dt + \sigma\,d\tilde{W}(t), \quad S(0) = S_0$$

Derive the explicit analytical expression for the risk-neutral probability of the following events:

(a)

$$\{K_1 < S(T) < K_2\}$$

(b)

$$\{S(T_2) > S(T_1) > S(t)\}, \quad \text{with } T_2 > T_1 > t > 0.$$

2. Assume the standard Black-Scholes model for a stock price as in Problem 1, A European *call spread* has payoff

$$\Lambda(S(T)) = \begin{cases} 0 & \text{if } S(T) \le K \\ S(T) - K & \text{if } K < S(T) < K + D \\ D & \text{if } S(T) \ge K + D \end{cases}$$

where $K > 0$ and $D > 0$ are given constants.

(a) Derive the formula for the option value $V(t, S)$ and delta $\Delta(t, S)$ at time $t$, in terms of the spot stock price $S(t) = S$, and time to maturity $T - t$.

(b) Find the limits of $V(t, S)$ as $D \to 0$ and $D \to \infty$, and give a financial interpretation for your answers.