Homework Assignment No. 3, due Monday, 2/6 at 5 pm

1. Use Itô's formula to show that
$$\int_0^t W(s)\,dW(s) = \frac{1}{2}W^2(t) - \frac{1}{2}t$$

2. Let $X(t) = e^{\sigma W(t)}$. Use Itô's formula to write down a stochastic differential for X. Then, by taking the expectation, find a first order linear ODE for $m(t) = \mathbb{E}[X(t)]$ and solve it to show that
$$\mathbb{E}\left[e^{\sigma W(t)}\right] = e^{\frac{\sigma^2}{2}t}$$

3. Two standard Brownian motions $dW^{(1)}(t)$ and $dW^{(2)}(t)$ are correlated with correlation ρ if
$$\mathbb{E}\left[dW^{(1)} \cdot dW^{(2)}\right] = \rho\,dt$$
Suppose that $Y(t)$ is driven by two correlated Brownian motions:
$$dY(t) = \mu\,dt + \sigma_1 dW^{(1)}(t) + \sigma_1 dW^{(2)}(t).$$
Show that alternatively we can express
$$dY(t) = \mu\,dt + \sigma dW(t)$$
where $W(t)$ is another standard Brownian motion. Derive the formula for σ in terms of σ_1, σ_2 and ρ.