1. Use Itô’s formula to show that
\[ \int_0^t W(s) \, dW(s) = \frac{1}{2} W^2(t) - \frac{1}{2} t \]

2. Let \( X(t) = e^{\sigma W(t)} \). Use Itô’s formula to write down a stochastic differential for \( X \). Then, by taking the expectation, find a first order linear ODE for \( m(t) = \mathbb{E}[X(t)] \) and solve it to show that
\[ \mathbb{E}[e^{\sigma W(t)}] = e^{\frac{\sigma^2}{2} t} \]

3. Two standard Brownian motions \( dW^{(1)}(t) \) and \( dW^{(2)}(t) \) are correlated with correlation \( \rho \) if
\[ \mathbb{E}[dW^{(1)} \cdot dW^{(2)}] = \rho \, dt \]
Suppose that \( Y(t) \) is driven by two correlated Brownian motions:
\[ dY(t) = \mu \, dt + \sigma_1 dW^{(1)}(t) + \sigma_1 dW^{(2)}(t). \]
Show that alternatively we can express
\[ dY(t) = \mu \, dt + \sigma dW(t) \]
where \( W(t) \) is another standard Brownian motion. Derive the formula for \( \sigma \) in terms of \( \sigma_1, \sigma_2 \) and \( \rho \).