Homework Assignment No. 2, due Wed, 1/25 at 5 pm

1. Consider the scaled symmetric random walk

$$W^{(n)}(t) = \frac{1}{\sqrt{n}} M_{\lfloor nt \rfloor}, \quad t \geq 0$$

   where M_k is the symmetric random walk.

   (a) Plot two sample paths $W^{(5)}(t)$ for $0 \leq t \leq 2$, mark all the discontinuities of the path.

   (b) Show that the distribution of $W^{(n)}(t)$ converges to a normal distribution with mean 0 and variance t, as $n \to \infty$;

   (c) For all $0 \leq s \leq t$ such that ns and nt are integers, show that

$$\mathbb{E}\left[W^{(n)}(t) | W^{(n)}(s) = x\right] = x;$$

   and

$$\mathrm{Var}\left(W^{(n)}(t)\right) = t, \quad \mathrm{Cov}\left(W^{(n)}(t), W^{(n)}(s)\right) = s.$$

2. Show that the following processes are standard Brownian motions.

   (a)

$$X(t) = W(T+t) - W(T), \text{ where } T > 0 \text{ is a constant.}$$

   (b)

$$X(t) = c\,W\left(\frac{t}{c^2}\right), \text{ where } c \neq 0 \text{ is a constant.}$$