1. Consider the scaled symmetric random walk

\[ W^{(n)}(t) = \frac{1}{\sqrt{n}} M_{\lfloor nt \rfloor}, \quad t \geq 0 \]

where \( M_k \) is the symmetric random walk.

(a) Plot two sample paths \( W^{(5)}(t) \) for \( 0 \leq t \leq 2 \), mark all the discontinuities of the path.

(b) Show that the distribution of \( W^{(n)}(t) \) converges to a normal distribution with mean 0 and variance \( t \), as \( n \to \infty \);

(c) For all \( 0 \leq s \leq t \) such that \( ns \) and \( nt \) are integers, show that

\[ \mathbb{E} \left[ W^{(n)}(t) | W^{(n)}(s) = x \right] = x; \]

and

\[ \text{Var} \left( W^{(n)}(t) \right) = t, \quad \text{Cov} \left( W^{(n)}(t), W^{(n)}(s) \right) = s. \]

2. Show that the following processes are standard Brownian motions.

(a) \[ X(t) = W(T + t) - W(T), \quad \text{where } T > 0 \text{ is a constant.} \]

(b) \[ X(t) = c W \left( \frac{t}{c^2} \right), \quad \text{where } c \neq 0 \text{ is a constant.} \]