1. An investor sells a European put on a share for $3. The stock price is $42 and the strike price is $40. Under what circumstances does the investor lose money? Under what circumstances will the option be exercised? Draw a diagram showing the variation of the investor’s profit/loss with the stock price at the maturity of the option.

2. Construct a 5-period binomial model to price a call and a put European stock options with strike price \( K = $50 \) and maturity \( T = 0.5 \), while the current stock price is assumed to be \( S_0 = $50 \), with volatility \( \sigma = 25\% \). The risk-free interest rate is assumed to be \( r = 2\% \). Compare these prices with the Black-Scholes formula prices and verify the put-call parity relation.

3. Price a treasury bond that matures in 5 years, with coupon rate \( c = 2.5\% \) that is paid semiannually. Also, obtain the yield for this bond. It is observed that the one-year zero-coupon bond is priced at $99 and the 5-year zero-coupon bond is priced at $90.48. Hint: you can assume that the current interest rate term structure to have a piecewise linear form that is based on the two data points from the above zero-coupon bond prices, and the assumption that the short interest rate is zero.

4. Consider

\[
X_n = \sum_{j=1}^{n} Z_j
\]

where

\[
Z_j = \begin{cases} 
1 & \omega = H \\
-1 & \omega = T 
\end{cases}
\]

with a fair coin toss, and all tosses are assumed to be independent. Show that \( \{X_n, n = 1, \ldots\} \) is a martingale.