Homework Assignment No. 1, due Wed, 1/18 at 5 pm

1. An investor sells a European put on a share for \$3. The stock price is \$42 and the strike price is \$40. Under what circumstances does the investor lose money? Under what circumstances will the option be exercised? Draw a diagram showing the variation of the investor's profit/loss with the stock price at the maturity of the option.

2. Construct a 5-period binomial model to price a call and a put European stock options with strike price $K = \$50$ and maturity $T = 0.5$, while the current stock price is assumed to be $S_0 = \$50$, with volatility $\sigma = 25\%$. The risk-free interest rate is assumed to be $r = 2\%$. Compare these prices with the Black-Scholes formula prices and verify the put-call parity relation.

3. Price a treasury bond that matures in 5 years, with coupon rate $c = 2.5\%$ that is paid semiannually. Also, obtain the yield for this bond. It is observed that the one-year zero-coupon bond is priced at \$99 and the 5-year zero-coupon bond is priced at \$90.48. Hint: you can assume that the current interest rate term structure to have a piecewise linear form that is based on the two data points from the above zero-coupon bond prices, and the assumption that the short interest rate is zero.

4. Consider
$$X_n = \sum_{j=1}^{n} Z_j$$
where
$$Z_j = \begin{cases} 1 & \omega = H \\ -1 & \omega = T \end{cases}$$
with a fair coin toss, and all tosses are assumed to be independent. Show that $\{X_n, n = 1, \ldots\}$ is a martingale.