Solutions to Sample Questions

1. 

\[ \sigma = \sqrt{\frac{(u - 1 - r\Delta t)(1 + r\Delta t - d)}{\Delta t}} \]

Here \( r \) is the annualized risk-free interest rate and \( \Delta t \) is measured in years. Since this is only for one period, using a generic \( r \) to denote the interest rate over the period is OK, which will leave just \( r \), instead of \( r\Delta t \) in the numerator.

2. If we use the approximation \( e^x \approx 1 + x \) for small \( x \), we get \( u = 1.1 \) and \( d = 0.9 \) (note that \( u-d \) is no longer 1 here). So the risk-neutral probabilities \( \hat{p} = \hat{q} = 1/2 \). The stock price at the end of period two can take \( S(HH) = 60.5 \), \( S(HT) = 49.5 \), and \( S(TT) = 40.5 \). The payoff of the call is 

\[ V(HH) = 8.5, \; V(HT) = V(TH) = 0. \]

Following the backward iteration steps, we have \( V_0 = 2.125 \). If you use the original formula \( u = e^{0.1} \approx 1.1052 \), the arithmetic would be a little more work but you should end up with just a slightly different answer.

3. In this case \( u = 1.2 \), \( d = 0.8 \), and \( \hat{p}, \hat{q} \) stay at \( 1/2 \), so \( S(HH) = 72 \), \( S(HT) = S(TH) = 48 \), and \( S(TT) = 32 \). The payoff of the put is 

\[ V(HH) = 0, \; V(HT) = V(TH) = 4, \; V(TT) = 20. \]

The put price at time 0 is \( V_0 = 7 \).

4. Let \( V \) denote the price of the contract of one share. We have 

\[ V(HH) = 8.5, \; V(HT) = V(TH) = -2.5, \; V(TT) = -11.5. \]

So \( V_0 = -2 \). If you enter this contract, you should get paid in an amount of \( 2 \times 100 = $200 \). To justify this answer, we consider the replicating portfolio where we borrow $52 from a bank to buy one share of the stock at the price $50, so we pocket $2 upfront. By the time \( T = 0.5 \), the payoff of this portfolio is \( S_T - K \), which is exactly the same as what you would get from this forward contract. Therefore you should get paid $200 to enter this contract for 100 shares.