1. This is an exercise to understand alternative, but equivalent ways to represent the information contained in a yield curve. Let us imagine a yield curve expressed through the zero rate \( r_0(T) = 0.005 + \alpha T + \beta T^2, \ 0 \leq T \leq 30 \) where we pick parameters \( \alpha = 0.002 \) and \( \beta = -0.00003 \). We pretend that data points for \( r_0 \) are only available at \( T_n \). In another word, only \( r_{0,n} = r_0(T_n) \) evaluated from the above function, are given to us for \( n = 0, 1, \ldots, N \), with \( N = 120 \) and \( \Delta t = 0.25 \).

(a) Use the data set \( \{r_{0,n}, n = 0, \ldots, N\} \) (not the original \( r_0 \) function) to obtain zero-coupon bond prices \( B_{0,n} \), for \( n = 0, \ldots, N \);
(b) Obtain the forward rates \( f_{0,n} \), for \( n = 0, \ldots, N - 1 \);
(c) Next we compute the price of a zero-coupon bond that matures in 4 years and 7 months. We can obtain an approximation based on the values of \( B_{0,18} \) and \( B_{0,19} \) by linear interpolation, or we can approximate \( r_0 \) for \( T = 55/12 \), based on \( r_{0,18} \) and \( r_{0,19} \), and then use the relation between the zero-coupon bond price and the zero rate to obtain an approximation for the bond price. Compare your approximations with the exact price \( \exp \left( -r_0(55/12) \frac{55}{12} \right) \). Repeat your approximations for another maturity 10 years and 2 months.

2. Suppose a coupon bearing bond with face value $100, coupons \( C \), paid semiannually, can be priced today as follows.

\[
P_0 = \sum_{i=1}^{N} \frac{C}{2} B_0(T_i) + 100B_0(T_N),
\]

where \( B_0(T) \) is the current price of a zero-coupon bond that matures at \( T \), and \( T_i, i = 1, \ldots, N \) are the coupon cash flow dates. The yield of the bond (\( y \)) is defined through

\[
P_0 = \sum_{i=1}^{N} \frac{C}{2} e^{-yT_i} + 100e^{-yT_N}.
\]

Compute the yield of a 10-year, 5% coupon bond based on the yield curve in Problem 1.

3. Problem 6.3 from the textbook.