Homework Assignment 4, Math 5760, due Oct 5 at 5 pm.

1. Implement the multi-period binomial model to price European calls and puts, using the following choices for parameters:

\[ u = e^{\sigma \sqrt{\Delta t}}, \quad d = e^{-\sigma \sqrt{\Delta t}}, \]

\[ \hat{p} = \frac{1}{2} \left( 1 + \left( \frac{r}{\sigma} - \frac{\sigma}{2} \right) \sqrt{\Delta t} \right), \quad \hat{q} = \frac{1}{2} \left( 1 - \left( \frac{r}{\sigma} - \frac{\sigma}{2} \right) \sqrt{\Delta t} \right). \]

The payoff function \( F(S) \) should be an input so the final payoff of the derivative can be calculated as \( V_N(S_N, N) = F(S_N) \) once the underlying stock price \( S_N \) at time \( N \) is revealed. Your program should take the following inputs: the current stock price \( S_0 \), the strike \( K \) of the option, the volatility \( \sigma \), the expiration \( T \), the number of periods \( N \) in calculation, and a flag to denote if it is a call or put.

2. The ETF SPY seeks to track the performance of S&P 500 index by holding all of the S&P 500 index stocks, while the leveraged ETF SSO claims to double the daily returns of S&P 500 index. Each of the ETFs has many call and put options written on it. Use your pricing model to estimate the implied volatilities of each underlying for 10 most actively traded options based trading volume, and plot them as functions of the strike (calls and puts should be plotted separately). Observe the relationship between the implied volatility curves for SPY and SSO, and make some comments.