Homework Assignment No.11, Due Tuesday, Dec 10 at 5 pm

1. Consider an European derivative with the payoff function

$$F(S_T) = \begin{cases} D - |S_T - K|, & \text{if } |S_T - K| \le D \\ 0, & \text{if } |S_T - K| > D \end{cases}$$

where $K > D > 0$ are given constants. This is an example of butterfly spread options, and it is commonly used as an option strategy.

   (a) Discuss the features of this derivative and suggest scenarios where certain investors may find this derivative particularly appealing.

   (b) Show that this derivative can be decomposed as a collection of vanilla calls or puts.

   (c) Use the Black-Scholes model to price this derivative.

2. One particular choice of parameters for the trinomial tree model for the stock price S

$$S_1 = \begin{cases} S_0 u & \text{with } p_u \\ S_0 & \text{with } p_m \\ S_0 d & \text{with } p_d \end{cases}$$

is the following:

$$u = e^{\sigma\sqrt{3\Delta t}}, \; d = 1/u,$$

$$p_d = \frac{1}{6} - \sqrt{\frac{\Delta t}{12\sigma^2}}\left(r - \frac{1}{2}\sigma^2\right), \; p_m = \frac{2}{3}, \; p_u = \frac{1}{6} + \sqrt{\frac{\Delta t}{12\sigma^2}}\left(r - \frac{1}{2}\sigma^2\right).$$

Show that this model satisfies the requirements:

$$\mathrm{E}\left[\frac{S_1 - S_0}{S_0}\right] \approx \left(r - \frac{1}{2}\sigma^2\right)\Delta t, \quad \mathrm{Var}\left[\frac{S_1 - S_0}{S_0}\right] \approx \sigma^2 \Delta t.$$

Here the approximation is defined as to the leading order of Δt, meaning that terms with higher powers of Δt can be ignored.

3. Use the random number generator (`rand` in both Matlab and Excel) to produce 12 draws of independent pseudo random numbers, uniformly distributed between 0 and 1. Convert them to 12 pseudo random numbers with standard normal distribution by using the Box-Muller algorithm (see the class note). What would these draws become after antithetic sampling and second moment matching? In antithetic sampling, once X is drawn from the distribution, $-X$ is also taken for the sample, therefore n draws will result in a sample of size $2n$. To match the second moment in a simulation, we simply make an adjustment by multiply all the draws by the same constant so that the sample variance of the sample matches exactly the variance of the distribution to be simulated.