

## Homework Assignment No.3, Due Tuesday, Sept. 24 at 5 pm

1. In this problem we consider two stocks X and Y, each has two possible outcomes tomorrow. There is no interest rate in this problem.
  - (a) Stocks X and Y are both traded at \$100 today. Stock X will be worth 110 tomorrow with probability 0.9 and 90 otherwise, and stock Y will be worth 110 tomorrow with probability 0.1 and 90 otherwise. Suppose both stocks have call options with the same strike 100. Which call is more valuable? Why or why not?
  - (b) There are only two possible states tomorrow: in state A, X will be 110 and Y will be 120, in state B, X will be 90 and Y will be 80. In each of the following cases, suggest a portfolio that will be riskless:
    - i. Using stocks X and Y, and zero-coupon bonds;
    - ii. Using the call option on X and the call option on Y, both struck at 100, and zero-coupon bonds;
    - iii. Using stock X and call option on Y with strike 100, and zero-coupon bonds;
    - iv. Using a call option on X with strike 100 and another call option on X with strike 105, and zero-coupon bonds.
2. Problem 3.3 from the text.
3.
  - (a) Again we assume no interest rate. Suppose the stock is traded at 50 today, and for each time step the stock price can either go up by 10 with probability  $p$ , or go down by 10 with probability  $1 - p$ . After 5 time steps, derive the probability distribution of the stock price. Compute the no-arbitrage prices of the at-the-money call and put options with the expiration at the end of 5 steps.
  - (b) Comment on the feasibility of the above model. What will happen if we need to compute options with expirations in more than 5 steps? Now consider the following modification: for each step, instead of going up or down by 10, the stock price will be multiplied by either 1.2 (with probability  $p$ ), or 0.8 (with probability  $1 - p$ ). Repeat the pricing for the at-the-money options using this model and compare with the prices in part (a).