Homework Assignment No.2, Due Tuesday, Sept. 17 at 5 pm

1. (Arbitrage) The XYZ stock is currently traded at \$100 per share, and suppose the one-year forward price at this time is also \$100. That is, it costs nothing to enter a contract where one party (the long position party) is committed to purchase a share of the stock one year later at the fixed price of \$100 from the counter party. The short position party (the counter party) is therefore committed to sell the long position party the share at this price. We assume that the current one-year risk-free interest rate is 1%.

   (a) Show that there is an arbitrage if we take the following positions: 1) short one share of the stock and deposit the proceed (\$100) in a bank at the risk-free rate; 2) take a long position in the aforementioned forward contract to buy one share of the stock at the end of one year for a price $K = \$100$. Demonstrate the arbitrage by showing that the initial cost of constructing this portfolio is zero, but a gain of \$1 is guaranteed, no matter what happens to the stock price at the end of one year. You can use simple compounding for this example.

   (b) In case investors are very optimistic about XYZ and a forward price $K = \$105$ appears, construct another arbitrage that takes advantage of this overestimated forward price.

   (c) Based the above two incorrect forward prices, prove that the correct K to be entered in the fair forward contract (implying zero value at the time of signing), the forward price, should be \$101.

2. (Zero-coupon Bond) A zero-coupon bond with face value (principal) \$1 will be traded on the market with a price less than \$1 any time before the maturity. The yield y is related to the price Z through

$$Z = e^{-yT}$$

   where T is the time to maturity, measured in years. The yield measures the return of the investment, if the bond is held till the maturity, and it fluctuates in response to the market. Imagine that the current bond has 5 years towards the maturity, and the current 5-year yield is 3%. If the 3-year interest rate at the end of the second year shoots to 6%, show that the bond holder could lose money if he/she decides to sell the bond at that time.

3. (Rational Bound) Exercise 2.16 from the text.

Suggested exercises to go over (no need to hand in): 2.3, 2.4, 2.8, 2.12.