

Homework Assignment No.1, Due Tuesday Sept. 10 at 5 pm

1. (Leverage) In this exercise, we consider a stock price in a one-period model, with beginning price S_0 , and end of the period price

$$S_1 = S_0(1 + \alpha + X)$$

where X is a random variable with mean zero and variance σ^2 , and $\alpha > 0$ is the expected growth of the stock. The return of the stock over this period of time is

$$R = \frac{S_1 - S_0}{S_0} = \alpha + X$$

so the expected return (growth) is α and the variance of return is σ^2 .

Instead of investing S_0 in the stock by buying one share of the stock, we can borrow some money to invest in γ shares of the stock. The portfolio then consists of γ shares of the stock, and a debt with the principal $Q = (\gamma - 1)S_0$.

- (a) Show that the return of the portfolio at the end of the period is

$$\frac{\Delta P}{P_0} = \frac{P_1 - P_0}{P_0} = \gamma(\alpha + X) - (\gamma - 1)R$$

where R is the interest rate over this period of time. Compute the expected return and the variance of the return, and then compare with those of the previous investment (with just one share in the stock).

- (b) Suppose $\alpha = 5\%$, $\gamma = 10$, $R = 2\%$, and X is a random variable with two possible values $X = 10\%$, or -10% , each with probability 0.5. Compute the loss (in percentage of the initial investment) for both the unleveraged and the leveraged cases.
2. (Hedging Portfolio) In continuation of Problem 1, we have two individual stocks, both begin with a price of \$1, and by the end of the period their prices are

$$S_1 = 1 + \alpha + X, \quad S_2 = 1 + \alpha + Y$$

where X and Y are two random variables with mean zero, and X has variance σ_1^2 . We assume two stocks are negatively correlated:

$$Y = \beta X + Z, \quad -1 < \beta < 0$$

and Z and X are independent from each other. The variance of Z is denoted by σ_2^2 .

- (a) We now construct a hedging portfolio with one share of S_1 and γ shares of S_2 , such that $1 + \gamma\beta = 0$. Show that the return of the hedging portfolio is

$$\frac{P_1 - P_0}{P_0} = \alpha + \frac{\gamma}{1 + \gamma}Z,$$

and the variance of the return is less than σ_2^2 .

- (b) Assume $\alpha = 5\%$, $\beta = -0.5$, $\sigma_1 = 0.25$, and $\sigma_2 = 0.2$, compute the variance of the return of the hedging portfolio.