Math 5760 Practice Problem Keys

1. Suppose you include β shares of stock and \$1 in loan,

$$P_0 = \beta S_0 - 1$$

and by the end of the investment period,

$$P_1 = \beta S_0(1+X) - (1+R)$$

$$\mathrm{E}[return] = \frac{\alpha\beta S_0 - R}{\beta S_0 - 1} = \mu, \quad \mathrm{Var}[return] = \frac{\sigma^2}{1 - \frac{\mu-\alpha}{\mu-R}}$$

2. (a) almost no arbitrage
   (b) One possible arbitrage portfolio: long call + short put + short S + deposit
   (c) One possible arbitrage portfolio: short K=45 call, long K=50 call, deposit \$5.

3. (a) digital call
   - $S = 100, Z = 1 : 0 \le C \le 1$
   - $S = 90, Z = 1 : 0 \le C \le 0.9$
   - $S = 110, Z = 0.9 : 0 \le C \le 0.9$
   - $S = 110, Z = 1 : 0 \le C \le 1$

   (b) portfolio
   - $S = 100, Z = 1 : 0 \le C \le 100$
   - $S = 90, Z = 1 : 0 \le C \le 90$
   - $S = 110, Z = 0.9 : 11.5 \le C \le 110$
   - $S = 110, Z = 1 : 0.5 \le C \le 110$

4.

$$p = 0.35, \sigma \approx 0.98, \quad \text{put price} \approx 1.9, \quad \Delta = -0.2$$

5.

$$p = 0.5, \quad C = P = 15.625.$$

6.

$$C = 3.8$$

7.

$$\frac{dS_t^2}{S_t^2} = (2r + \sigma^2)\,dt + 2\sigma\,dW_t$$

$$C = \frac{e^{-rT}}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \max\left(S_0^2 \exp\left((2r - \sigma^2)T + 2\sigma\sqrt{T}x\right) - K, 0\right) e^{-\frac{x^2}{2}}\,dx$$

8.

$$P = e^{-rT}\left(\log S_0 + (r - \frac{1}{2}\sigma^2)T\right)$$

9. (refers to problem 4)

$$p = 0.957, \sigma_{imp} \approx 118\%$$

10.

$$\sigma_1 = 0.1,\ \sigma_2 = 0.187,\ \sigma_3 = 0.257$$

11.

$$P_{am} \approx 1.98$$

12.

$$C_{barrier} = 3.5$$