

Math 5760/6890 Project, due Oct. 23 at 5 p.m.

1. Implement the multiple-step binomial model to price European calls/puts, and American puts, using the following choices for parameters:

$$\mu = 0, \quad u = e^{\sigma\sqrt{\Delta t}}, \quad d = e^{-\sigma\sqrt{\Delta t}},$$

$$p = \frac{1}{2} \left(1 + \left(\frac{r}{\sigma} - \frac{\sigma}{2} \right) \sqrt{\Delta t} \right)$$

The payoff function $F(S)$ should be left as a user-supplied function, so the final payoff of the derivative can be calculated as $V_N(S(T)) = F(S(T))$ once the underlying stock price $S(T)$ at time $T = t_N$ is revealed. Your program should take the following inputs: current stock price $S(0)$, strike K , expiration T , volatility σ , risk-free interest rate r , number of time steps N , a flag to denote if it is a call or put, and another flag to indicate whether it is a European or American style if the option is a put. Test your program thoroughly by comparing test results with the Black-Scholes formula.

2. Download the daily prices for the index S&P 500 (SPX) and the ETF ProShare Ultra S&P 500 (SSO) from finance.yahoo.com, for the period beginning 1/3/2012 till now.
 - (a) Compute the daily percentage returns and log returns based on day closes for both SPX and SSO, and compare them to see if SSO daily returns really double the daily returns of SPX as claimed. Also compare the year-to-date returns to see if SSO really doubles the SPX return in year-to-date form.
 - (b) The historical volatility computed at the close of day k , based on information from M previous trading days, can be estimated as

$$\tilde{\sigma}_k = \sqrt{\frac{252}{M-1} \left(\sum_{i=0}^{M-1} R_{k-i}^2 - \frac{1}{M} \left(\sum_{i=0}^{M-1} R_{k-i} \right)^2 \right)}$$

where

$$R_k = \frac{S_k - S_{k-1}}{S_{k-1}}$$

is the return on day k .

Use this formulas to estimate the time dependent historical volatility $\tilde{\sigma}_k$ for the last two months, and plot the estimates as a time series. Use $M = 10, 20$, and 30 business days separately for comparison. What conclusion can you draw regarding the correlation between historical volatility of SPX and the SPX index itself?

3. The ETF fund SPY seeks to track the performance of S&P 500 index by holding all of the S&P 500 index stocks, while the leveraged ETF fund SSO claims to double the daily returns of S&P 500 index. Each of the ETFs has many call and put options written on it. Use the Black-Scholes formula to estimate the implied volatilities of each underlying for 10 most actively traded options according to trading volume, and plot them as functions of the strike (calls and puts should be plotted separately). Observe the relationship between the implied volatility curves for SPY and SSO, and make some comments.