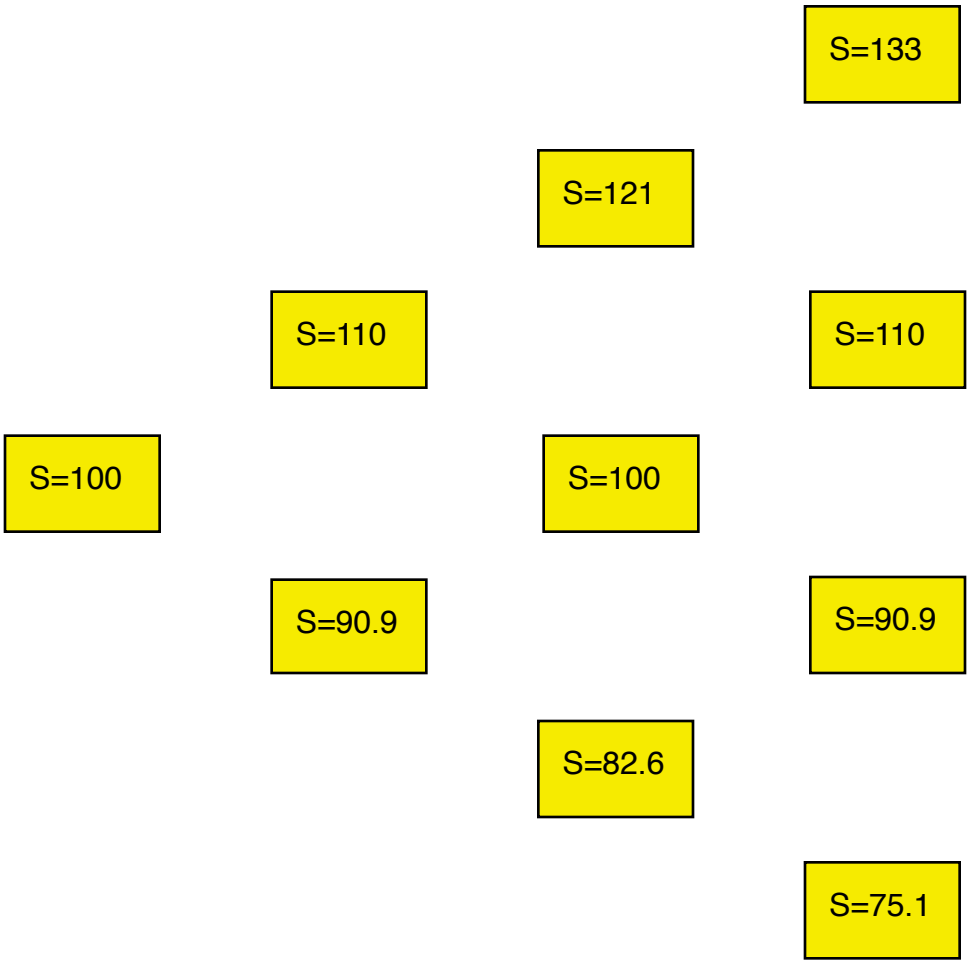
A Numerical Example of Pricing Options on a Multi-step Tree with Risk-Neutral Probabilities

Here we give an example that illustrates the use of backward iterations to price derivatives based on a multi-step binomial tree.

Suppose we have the following 3-step binomial tree with stock price specified on each node. Assume a 1% interest rate, and we calculate the risk-neutral probabilities for all the nodes to be the same values (approximately 0.53 for up moves and 0.47 for down moves).

In the following, we illustrate the steps to price a European put and an American put using the following diagrams.



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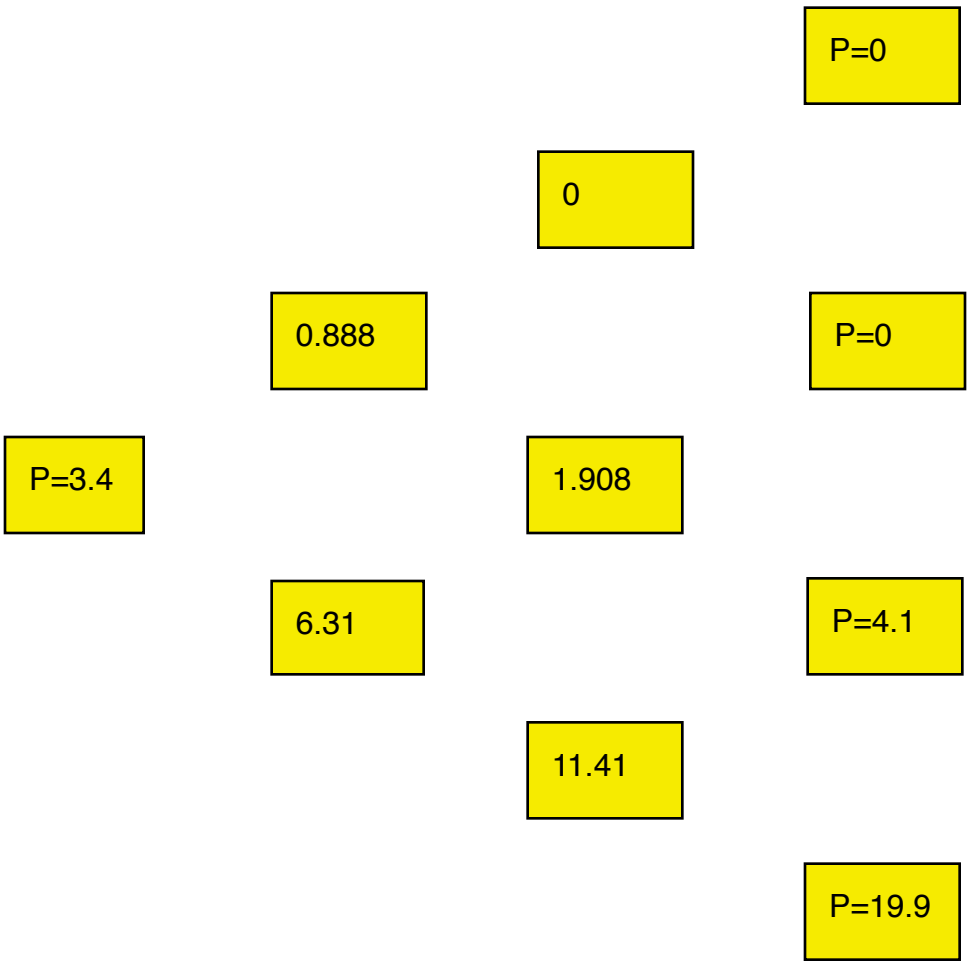
Suppose we want to price a European put with strike price K=$95 that expires at t=3. We start the iteration from the expiration time, where the payoff can be readily calculated for each state as P=max(K-S,0). For one step back (t=2), the value of the put at each node is the risk-neutral expectation of the put value at t=3, discounted based on the interest rate. For example, the value 11.41 is obtained as

(0.53*4.1 + 0.47*19.9)/1.01 = 11.41

Once we have the values for nodes at t=2, we can repeat the steps to obtain values for nodes at t=1. Eventually we get the value of the put at t=0 as $3.4.



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Next we use the same tree to price the corresponding American put with the same strike that can be exercised any time before or at t=3. Notice at t=3, we either give up on the put or just exercise it (with asterisk attached to the value) and the values are the same as the European case. However, when we back it one time to arrive at a discounted expected value, we are given the opportunity to exercise the put, it it yields more value. Again, look at the entry with 12.4*, the previous value (discounted expected value 11.41) is compared with the immediate exercise payoff 95-82.6=12.4, and the decision is to exercise it (so there is the asterisk attached) as the value of continuation is only 11.41. The process is repeated and we obtain the current American put price $3.62, which is more valuable than the European put.