Math 5610/6860, Project No.2

1. Problem 4.5.12 from the textbook. Write a MATLAB (or equivalent Maple) program to do the calculations. Notice that MATLAB uses double precision as its default arithmetic. To appreciate the accuracy brought by double precision, modify your code to run everything in single precision (using `single`) and compare the results with the double precision case.

2. Implement the adaptive quadrature algorithm 4.3 on page 216 to approximate the integrals in problem 4.6.4 to within 10^{-6}. Compare your results with those generated from the standard MATLAB function `quad`. You will need to learn how to use a function handle in calling this function in MATLAB.

3. Use the MATLAB function `dblquad` to approximate

$$\int_{-L}^{L}\int_{-L}^{L} e^{-x^2-y^2}\,dxdy$$

for $L = 1, 5$ and 10. You can set the tolerance to the default 10^{-6}.

4. Implement Algorithm 5.2 (Runge-Kutta of order 4) to solve equations in problem 5.4.2 from the textbook, using $h = 0.1$.

5. Use the MATLAB function `ode113` to solve the same problems in 5.4.2 with the same $h = 0.1$. Compare with the results from Runge-Kutta.

6. (Required only for those registered in 6860, extra credit with those in 5610)

The approximation of

$$\iiint_D f(x,y,z)\,g(x,y,z)\,dx\,dy\,dz$$

can be expensive with a conventional multiple quadrature rule for three space dimensions or higher dimensions. Here $g(x,y,z) > 0$ has the property that $\iiint g\,dx\,dy\,dz = 1$. One alternative is to view the integral as the expectation of f, where it depends on random variables X, Y, Z with a joint distribution $g(x,y,z)$. Then we make the following approximation

$$\mathbb{E}[f] \approx \frac{1}{N}\sum_{i=1}^{N} f(X_i, Y_i, Z_i)$$

by a sample $(X_i, Y_i, Z_i), i = 1, N$, drawn from the distribution with density g.

Use this technique to give an estimate of the integral

$$\frac{1}{(2\pi)^{3/2}}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \exp\left(x + y + z - \frac{1}{2}\left(x^2+y^2+z^2\right)\right) dx\,dy\,dz$$

and compare with the exact answer $e^{3/2}$ with $N = 100, 1000, 10000$, and 100000 points. The MATLAB function `randn` should be used to generate a sample of normally distributed random numbers.