Some Solution Notes for Assignment No.3

Here we address several confusing points encountered in this assignment.

- Problem 2.4.8

  Part (a). We need to show

$$\frac{10^{-2^{n+1}}}{(10^{-2^n})^2} = \frac{10^{-2^{n+1}}}{10^{-2\cdot 2^n}} = \frac{10^{-2^{n+1}}}{10^{-2^{n+1}}} = 1$$

  Part (b). Note that

$$\frac{10^{-(n+1)^k}}{\left(10^{-n^k}\right)^2} = 10^{2n^k-(n+1)^k},$$

  and

$$2n^k - (n+1)^k = 2n^k - \sum_{i=0}^{k} \binom{k}{i} n^i = 2n^k - \left(n^k + kn^{k-1} + \cdots\right) = n^k - kn^{k-1} - \cdots$$

  The above goes to infinity as $n \to \infty$ so

$$10^{2n^k-(n+1)^k} \to \infty$$

- Problem 2.5.14

  Part (b).

$$\frac{1/(n+1)^{n+1}}{1/n^n} = \frac{n^n}{(n+1)^{n+1}} = \left(\frac{n}{n+1}\right)^n \cdot \frac{1}{n+1}$$

  We are reminded the famous limit $\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n = e$, therefore the limit of the above as $n \to \infty$ is $e^{-1} \cdot 0 = 0$.

  On the other hand, for $\alpha > 1$,

$$\frac{1/(n+1)^{n+1}}{1/n^{\alpha n}} = \frac{n^{\alpha n}}{(n+1)^{n+1}} = \left(\frac{n}{n+1}\right)^n \frac{n^{(\alpha-1)n}}{n+1}.$$

  The first factor approaches e^{-1}, while the second factor approaches infinity, therefore the product has infinity as its limit.

- Problem 3.1.2

  For the linear interpolation, we only need two points and must make a decision as which two points should be used. If we want to interpolate to $x = 1.4$, we should use the data points $x = 1.25$ and $x = 1.6$. The data point $x = 1$ should be left alone.