Some Solution Notes for Assignment No.3

Here we address several confusing points encountered in this assignment.

- Problem 2.4.8
  Part (a). We need to show
  \[
  \frac{10^{-2^{n+1}}}{(10^{-2^n})^2} = \frac{10^{-2^{n+1}}}{10^{-2^2}} = \frac{10^{-2^{n+1}}}{10^{-2^{n+1}}} = 1
  \]
  Part (b). Note that
  \[
  \frac{10^{-(n+1)^k}}{(10^{-n})^2} = 10^{2n^k-(n+1)^k},
  \]
  and
  \[
  2n^k - (n+1)^k = 2n^k - \sum_{i=0}^{k} \binom{k}{i} n^i = 2n^k - (n^k + kn^{k-1} + \ldots) = n^k - kn^{k-1} - \ldots
  \]
  The above goes to infinity as \( n \to \infty \) so
  \[
  10^{2n^k-(n+1)^k} \to \infty
  \]

- Problem 2.5.14
  Part (b).
  \[
  \frac{1}{(n+1)^{n+1}} = \frac{n^n}{(n+1)^{n+1}} = \left( \frac{n}{n+1} \right)^n \frac{1}{n+1}
  \]
  We are reminded the famous limit \( \lim_{n \to \infty} \left(1 + \frac{1}{n}\right)^n = e \), therefore the limit of the above as \( n \to \infty \) is \( e^{-1} \cdot 0 = 0 \).
  On the other hand, for \( \alpha > 1 \),
  \[
  \frac{1/(n+1)^{n+1}}{1/n^{\alpha n}} = \frac{n^{\alpha n}}{(n+1)^{n+1}} = \left( \frac{n}{n+1} \right)^n \frac{n^{(\alpha-1)n}}{n+1}.
  \]
  The first factor approaches \( e^{-1} \), while the second factor approaches infinity, therefore the product has infinity as its limit.

- Problem 3.1.2
  For the linear interpolation, we only need two points and must make a decision as which two points should be used. If we want to interpolate to \( x = 1.4 \), we should use the data points \( x = 1.25 \) and \( x = 1.6 \). The data point \( x = 1 \) should be left alone.