Midterm 2 Practice Problems, Math 2280, Fall 2012

This exam will cover Sections 4.1-4.2, 5.1-5.6, and 6.1-6.2.

1. Rewrite the following third-order equation for $y(t)$ as a system of first-order equations for a vector $\mathbf{x}(t)$. In the matrix notation $\mathbf{x}' = A\mathbf{x} + \mathbf{f}(t)$, identify the matrix A and the forcing term $\mathbf{f}$.
$$y''' - 2y' + 3ty = \sin t$$

2. Find general solutions of the following linear system, using the method of elimination.
$$\begin{aligned} x' &= \quad x + 9y \\ y' &= -2x - 5y \end{aligned}$$

3. Let $x_1(t)$ and $x_2(t)$ denote the amounts of salt in the two brine tanks satisfying
$$\begin{aligned} \frac{dx_1}{dt} &= -2x_1 + x_2 \\ \frac{dx_2}{dt} &= 2x_1 - x_2 \end{aligned}$$
Use the eigenvalue method to solve this system for $x_1(0) = 5,\ x_2(0) = 10$, and sketch the graphs of $x_1(t)$ and $x_2(t)$.

4. Consider the nonhomogeneous system
$$\mathbf{x}' = A\mathbf{x} + \mathbf{f}$$
where
$$A = \begin{bmatrix} 2 & -1 \\ 0 & 2 \end{bmatrix}, \qquad \mathbf{f} = \begin{bmatrix} e^t \\ 0 \end{bmatrix}$$
   (a) Compute e^{At} by taking advantage of the fact that $A = 2I + B$ where B has only one nonzero entry.

   (b) Suppose $\mathbf{x}(0) = 0$, solve the initial value problem for the above linear system.

5. For the nonlinear system
$$\begin{aligned} x' &= 6x - 5y + x^2 \\ y' &= 2x - y + y^2 \end{aligned}$$
   (a) Linearize the system around the critical point $(0, 0)$;

   (b) Determine the type of stability of the almost linear system by analyzing the linearized system around $(0, 0)$.