Midterm 1 Practice Problems Answers, Math 2280, Fall 2012

1. (a)

$$y(x) = \frac{1}{4}(x-1)^4$$

(b)

$$x(t) = -\frac{1}{\sqrt{4-2t^2-2t^4}}$$

2. The equation for P is

$$\frac{dP}{dt} = (0.03 - 0.001P)P = 0.001P(30-P), \quad M = 30$$

The critical point $P = 30$ is stable.

3. As $t \to \infty$, $v(t) \to 0$ but x continues to grow.

4. (a)

$$r^2 + 2r + 5 = (r+1)^2 + 4 = 0, \quad r = -1 \pm 2i$$

$$y_c = e^{-x}(c_1 \cos 2x + c_2 \sin 2x), \quad y_p = \frac{9}{17}(\cos 2x + 4 \sin 2x)$$

$$y = \frac{1}{17}\left(9 + 8e^{-x}\right)\cos 2x + \frac{4}{17}\left(9 - 8e^{-x}\right)\sin 2x$$

(b)

$$r^3 + r = r(r^2+1) = 0, \quad r = 0,\ r = \pm i$$

$$y = c_1 + c_2 \cos x + c_3 \sin x$$

5. The critical damping coefficient $c_{cr} = \sqrt{4mk} = 6$, so the first case is an underdamped case, and the second case is an overdamped case.

(a)

$$x = e^{-t}\left(A\cos(2\sqrt{2}t) + B\sin(2\sqrt{2}t)\right)$$

(b)

$$x = c_1 e^{-(4+\sqrt{7})t} + c_2 e^{-(4-\sqrt{7})t}$$