Midterm 1 Practice Problems, Math 2280, Fall 2012

Instructions: The exam will be a closed book and closed note test. You need to show all the details of your work to receive full credit. Calculators are not allowed and we will make sure that numerical calculations involved are kept at minimum.

1. Solve the following differential equations with prescribed initial conditions.

   (a)
   $$\frac{dy}{dx} = (x-1)^3, \quad y(1) = 0$$

   (b)
   $$\frac{dx}{dt} = 2tx^3 + 4t^3x^3, \quad x(0) = -\frac{1}{2}$$

2. Suppose for a certain species the population $P(t)$ is described by a birth rate $\beta = 0.05 - 0.002P$, and a death rate $\delta = 0.02 - 0.001P$. Find the limiting population M without solving the corresponding differential equation. Determine if this critical point $P = M$ is stable or not. Why?

3. Consider the resistance model for a body moving through a medium with a velocity v : $dv/dt = -kv^2$. Show that
   $$v(t) = \frac{v_0}{1 + v_0kt}, \qquad x(t) = x_0 + \frac{1}{k}\log(1 + v_0kt).$$
   Discuss the behavior of velocity and distance when t approaches infinity. Does the object travel only a finite distance as the velocity becomes smaller and smaller?

4. Solve the following linear equation with constant coefficients. Find the solution that satisfies the initial conditions if they are given, otherwise just obtain a general solution.

   (a)
   $$y'' + 2y' + 5y = 9\cos 2x, \qquad y(0) = 1,\ y'(0) = 0.$$

   (b)
   $$y''' + y' = 0$$

5. For the free damped motion with $m = 1, k = 9$, and the resistance coefficient c given below, determine whether the system is underdamped, critically damped, or overdamped. Write down the form of the solution in both cases.

   (a) $c = 2$

   (b) $c = 8$