Solution Notes for Homework Assignment 5

3.3.2 two distinct real roots.

$$2r^2 - 3r = r(2r - 3) = 0$$

3.3.6 a pair of complex roots.

$$r^2 + 5r + 5 = 0$$

3.3.14 two distinct real roots, and a pair of imaginary roots.

$$r^4 + 3r^2 - 4 = (r^2 + 4)(r^2 - 1) = 0$$

3.3.18

$$r^4 - 16 = (r^2 + 4)(r^2 - 4) = 0$$

3.3.22

$$9r^2 + 6r + 4 = (3r + 1)^2 + 3 = 0, \ r = -\frac{1}{3} \pm \frac{1}{\sqrt{3}} i$$

3.3.25

$$3r^3 + 2r^2 = r^2(3r + 2) = 0,$$

we have $r = 0$ as a double root (corresponding to $c_1 + c_2 x$), and another $r = -2/3$. The general solution is

$$y = c_1 + c_2 x + c_3 e^{-2x/3}$$

We differentiate twice to have y' and y'', plug in $x = 0$ in all of them (including y) and set them to the corresponding initial values. Then we have three linear equations to solve for c_1, c_2 and c_3.

3.3.28

$$2r^3 - r^2 - 5r - 2 = 0$$

$r = 2$ is an obvious integral root. Use long division, we can factor it into another quadratic function multiplied to $r - 2$.

3.3.40 e^{2x} refers to $r = 2$, and the other two suggest $r = \pm 2i$.

3.3.52

$$\frac{dy}{dx} = \frac{dy}{dv}\frac{dv}{dx} = \frac{1}{x}\frac{dy}{dv},$$

$$\frac{d^2y}{dx^2} = \frac{d}{dx}\frac{dy}{dx} = \frac{d}{dx}\left(\frac{1}{x}\frac{dy}{dv}\right) = -\frac{1}{x^2}\frac{dy}{dv} + \frac{1}{x^2}\frac{d^2y}{dv^2}$$

so the equation in v reads

$$\frac{d^2y}{dv^2} + 9y = 0$$

3.4.2 Use the formula $T = 2\pi/\omega_0$, where $\omega_0 = \sqrt{k/m}$.

3.4.5 Equation (19) on page 105 suggests that when an object is near the surface of the earth, the acceleration will be GM/r^2, this should replace g in Equation (6) on page 187, so the circular frequency for a pendulum with length L located at a distance R form the earth center is

$$\sqrt{\frac{GM}{R^2L}}$$

Use the definition of period and its relation with frequency, we should be able to arrive at that formula.

3.4.9 Need to use $kx_0 = mg$

3.4.18 First determine how the system is damped. Compare c with $c_{cr} = \sqrt{4mk}$, then use the appropriate solution.

3.4.24 In the critically damped case, we have a double root, the general solutions is

$$x = (c_1 + c_2t)e^{-pt}$$

3.4.27 Here we have two distinct real roots.

3.5.2 Try $y_p = Ax + B$.

3.5.13 Try $y_p = e^x(A\cos x + B\sin x)$.

3.5.23 Since $\cos 2x$ and $\sin 2x$ satisfy the homogeneous equation, we need to include $x^2\cos 2x$ and $x^2\sin 2x$.

3.5.28 Similar to 3.5.23, $\sin 3x$ satisfies the homogeneous equation, we need to include terms with x^3 multiplied.

3.5.34 First try $y_p = x(A\cos x + B\sin x)$, determine A and B, then combine with $y_c = c_1\cos x + c_2\sin x$, choose c_1 and c_2 so that $y_c + y_p$ satisfies the initial conditions.

3.5.38 $\sin 3x$ does not satisfy the homogeneous equation so we can just try $y_p = A\sin 3x + B\cos 3x$.