Solution Notes for Homework Assignment 1

1.1.24

$$y(1) = 1 \cdot (C + \log 1) = C = 17$$

1.1.29 The slope of a line that is normal to the graph of $y = g$ is $-1/y'$. On the other hand, this slope is equal to $(y-1)/x$, so we must have

$$\frac{y-1}{x} = -\frac{1}{y'} \quad \text{or } y'(y-1) = x$$

The graph of g is a circle centered at $(0, 1)$.

1.1.45 From $dP/dt = 1 = 10^2 k$, we deduce $k = 0.01$. The solution is therefore

$$P = \frac{2}{1 - 0.02t}$$

Solve for $P(t) = 100$ we have $t = 49$.

1.2.3

$$y = \int \sqrt{x}\, dx = 2x^{3/2} + C, \quad y(4) = 16 + C = 0, \quad C = -16.$$

1.2.7

$$y = \int \frac{10}{1+x^2} dx = 10 \tan^{-1} x + C, \quad y(0) = C = 0, \quad y = 10 \tan^{-1} x$$

1.2.16

$$v = \int \frac{1}{\sqrt{t+4}} dt = 2\sqrt{t+4} + C, \quad v(0) = 4 + C = -1, \quad C = -5, \quad v(t) = 2\sqrt{t+4} - 5$$

1.2.24

$$y(t) = -\frac{1}{2} g t^2 + 400, \quad g = 32$$

Find t such that $y(t) = 0$ and then plug into $v(t) = -gt$ to get the velocity.

1.2.30

$$\frac{dv}{dt} = -k, \quad v(t) = v_0 - kt, \quad x(t) = 88t - \frac{1}{2} k t^2$$

At $t = v_0/k,\ x = 176$, we can solve for $k = 22\ ft/s^2$

1.3.12 $f = x \log y$, and $f_y = x/y$ are both continuous near $(1, 1)$.

1.3.30

$$y' = \begin{cases} 0 & \text{if } x \leq c, \\ -\sin(x - c) & \text{if } c < x < c + \pi, \\ 0 & \text{if } x \geq c + \pi \end{cases}$$

If $|b| > 1$, $1 - y^2(0) < 0$ so f is no longer defined. When $|b| < 1$, f and f_y are continuous near (a, b) so there is a unique solution. When $|b| = 1$, Theorem 1 does not apply, but we have infinitely many solutions as we can see $y = \cos(x - a)$ is a solution, and the piecewise functions defined in the problem can also be solutions.