

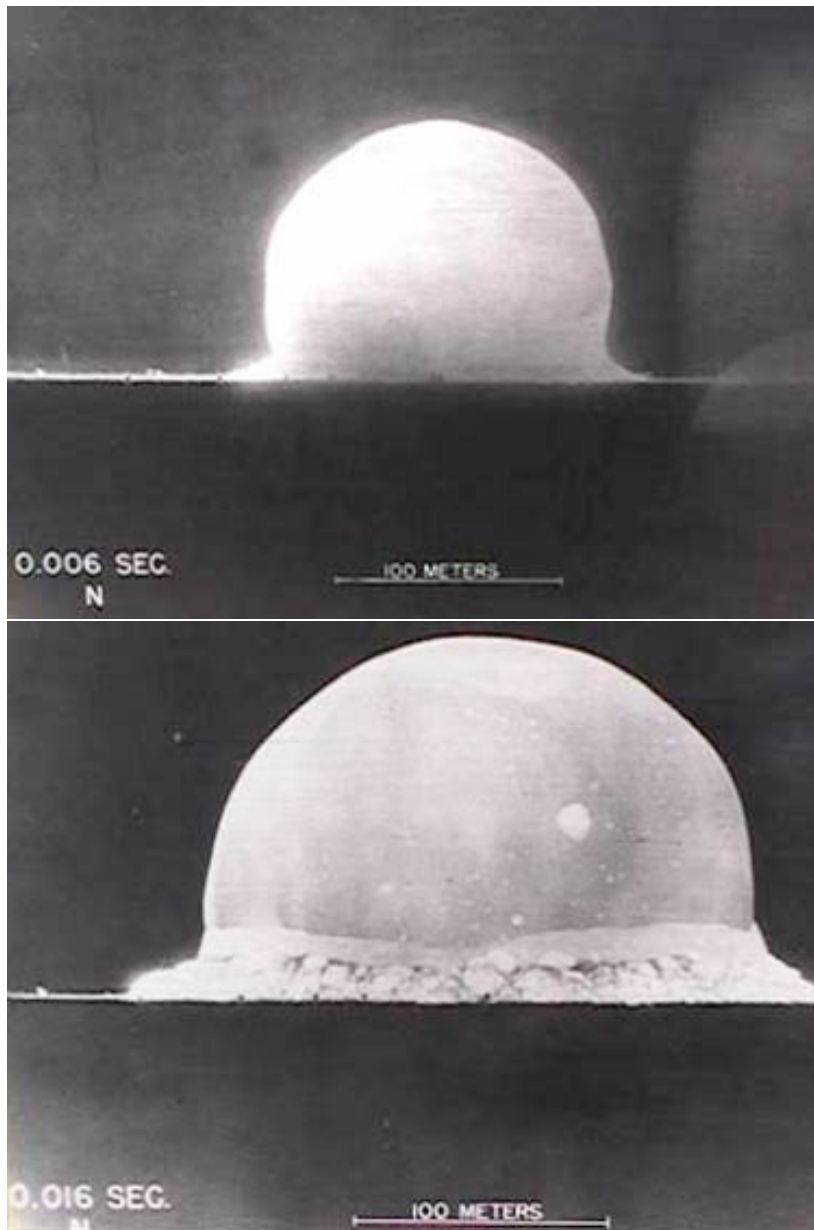
## Estimate of the energy released in the first Atomic Bomb explosion.

This document is adapted after the URL: <http://www.pa.uky.edu/~sps/Month1.htm> (snapshot as of Sept. 10, 2004).

My only contribution was typesetting the above mentioned web document in L<sup>A</sup>T<sub>E</sub>X.

The first explosion of an atomic bomb was the Trinity test in New Mexico in 1945. Several years later a series of pictures of the explosion, along with a size scale, and time stamps were released and published in a popular magazine. Based on these photographs a British physicist named G. I. Taylor was able to estimate the *power released by the explosion* (which was still a secret at that time).

How can the following pictures be used to make this estimate?





First two assumptions need to be made:

1. The energy ( $E$ ) was released in a small space.
2. The shock wave was spherical.

We have the size of the fire ball ( $R$  as a function of  $t$ ) at several different times. How does the radius ( $R$ ) depend on:

- energy ( $E$ )
- time ( $t$ )
- density of the surrounding medium ( $\rho$  – initial density of air)

Let's perform a dimensional analysis of the problem:

- $[R] = L$  :radius is determined by a distance
- $[E] = ML^2/T^2$  :energy is determined by a mass times a distance squared divided by time squared.
- $[t] = T$  :Time is determined by the time.
- $[\rho] = M/L^3$  :density is determined by a mass divided by a distance cubed.

We can say

$$[R] = L = [E]^x [\rho]^y [t]^z$$

Substituting the units for energy, time and density that we listed above we have:

$$[R] = L = M^{(x+y)} L^{(2x-3y)} T^{(-2x+z)}$$

$M$  is to the  $x + y$  power because energy and density are both dependent on  $M$ .

$L$  is to the  $2x - 3y$  power because energy is dependent on the square of distance and density is dependent on one over the cube of distance.

$T$  is to the  $-2x + z$  power because energy is dependent on one over the square of time and time is dependent on time. This provides three simultaneous equations:

$$\begin{aligned} x + y &= 0, \\ 2x - 3y &= 1, \\ -2x + z &= 0, \end{aligned}$$

yielding the results:

$$x = 1/5, \quad y = -1/5, \quad z = 2/5.$$

The radius of the shock wave is therefore:

$$R = E^{1/5} \rho^{-1/5} t^{2/5} * constant$$

Let's assume the constant is approximately 1.

Solving the equation for  $E$  we get:

$$E = (R^5 \rho) / t^2.$$

At  $t = .006$  seconds the radius of the shock wave was approximately 80 meters. The density of air is  $\rho = 1.2 \text{ kg}/\text{m}^3$ . Plugging these values into the energy equation gives:

$$\begin{aligned} E &= (80^5) \times 1.2 / (.0062) \text{ kg} * \text{m}^2 / \text{s}^2 \\ &= 1 \times 10^{14} \text{ kg} * \text{m}^2 / \text{s}^2 \\ &= 1 \times 10^{21} \text{ ergs} \end{aligned}$$

Now, 1 g of TNT =  $4 \times 10^{10} \text{ erg}$ , and hence

$$E = 25 \text{ kilo} - \text{tons of TNT}$$

We would like to thank Prof. Wolfgang Korsch for presenting this problem and its solution.