MATH 1210 MIDTERM EXAM PROBLEMS

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1. First Midterm

1. Find the limits. (If the limit does not exist, write "DNE".)
(a) $\lim_{x \to -2} |x+2|$
(b) $\lim_{x \to 2} \dfrac{x^2 - 5x + 6}{x - 2}$
(c) $\lim_{x \to 1^+} \sqrt{x-1}$
(d) $\lim_{x \to 1^-} \sqrt{x-1}$

2. Find the limits. (If the limit does not exist, write "DNE".)
(a) $\lim_{x \to -\infty} \dfrac{5x}{x^2 - 2x + 5} + \dfrac{14x^2 + 11}{(2x-8)^2}$.
(b) $\lim_{x \to -\infty} \sqrt{x^2 + 7x - 6} + x$.

3. If $\sin(x) = -\frac{7}{9}$ where $-\pi < x < -\frac{\pi}{2}$, find the values of the following trigonometric functions. You may use the following formulae:

$$\sin(2x) = 2\sin(x)\cos(x), \qquad \cos(\frac{x}{2}) = \pm\sqrt{\frac{1+\cos(x)}{2}}.$$

(a) $\sin(2x)$
(b) $\cos(\frac{x}{2})$.

4. Determine if the *Intermediate Value Theorem* implies that the equation $3x^{101} - (x+1)\cos(x) = 0$ has a solution in the interval $(-1, 0)$. Explain your answer.

5. Let $f(x) = \sqrt{x}$. Write down the definition (using limit) of $f'(x)$. Then find $f'(x)$ using the definition.

2. Second Midterm

1. Find the limit $\lim_{h \to 0} \dfrac{(x+h)^{100} - x^{100}}{h}$. (Hint: The limit represents the derivative of a function.)

2. Find derivatives: $\frac{dy}{dx}$
(a) $y = \cos(\sin(x))$.
(b) $y = (x^2+1)^8(-8x)^{-7}$

3. Find the equation of the tangent line to the curve

$$xy^2 - 10xy + 21 = 0$$

at $y = 3$.

4. The top of a 18 foot ladder, leaning against a vertical wall, is slipping down the wall at the rate of 2 feet per second. How fast is the bottom of the ladder sliding along the ground when the bottom of the ladder is 6 feet away from the base of the wall?

5. Let $f(x) = 5x^2 - 3x + 2$. Explain that $f(x)$ has an absolute maximum and an absolute minimum on the interval $[0, 5]$ and find them.

3. Third Midterm

1. Let $f(x) = (x+5)^2(x-6)^3$.
(a) Find all critical points of f.
(b) Indicate where $f(x)$ is increasing and where $f(x)$ is decreasing.
(c) Find the x-coordinates of all local maxima and minima.

2. A cylinder is inscribed in a right circular cone of height 2 and radius (at the base) equal to 1. What are the dimensions of such a cylinder which has maximum volume?

3. Let $f(x) = \pi \sin(\pi x)$.
(a) $|f'(x)| \leq k_1$, where k_1 is a real number. Find k_1.
(b) By Mean Value Theorem, $|f(b) - f(a)| \leq k_2|b-a|$, where k_2 is a real number. Find k_2.
(c) By mean value theorem, we know that there exists at least one c in the interval $(0, 2)$ such that $f'(c)$ is equal to the mean slope. Find all values of c.

4. Find $\dfrac{d}{dx}\displaystyle\int_{t=-5}^{\sqrt{x}} \frac{\cos(t)}{t^{11}} dt$.

5. A particle is moving with acceleration $a(t) = 6t + 2$. Its position at time $t = 0$ is $s(0) = 3$ and its velocity at time $t = 0$ is $v(0) = 2$. What is its position at time $t = 1$?

6. Calculate $\displaystyle\int_{x=0}^{4} \left(x^2 + 3\right) dx$ by Riemann sum $\displaystyle\lim_{n\to\infty} R_n$. You may need the formula $\displaystyle\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$.
(a) Find R_n using right end points.
(b) Find $\lim_{n\to\infty} R_n$.