1. First Midterm

1. Find the limits. (If the limit does not exist, write “DNE”.)
   (a) \( \lim_{x \to -2} |x + 2| \)
   (b) \( \lim_{x \to 2} \frac{x^2 - 5x + 6}{x - 2} \)
   (c) \( \lim_{x \to 1^+} \sqrt{x - 1} \)
   (d) \( \lim_{x \to 1^-} \sqrt{x - 1} \)

2. Find the limits. (If the limit does not exist, write “DNE”.)
   (a) \( \lim_{x \to -\infty} \frac{5x}{x^2 - 2x + 5 + (2x - 8)^2} \)
   (b) \( \lim_{x \to -\infty} \sqrt{x^2 + 7x - 6 + x} \)

3. If \( \sin(x) = -\frac{7}{9} \) where \( -\pi < x < -\frac{\pi}{2} \), find the values of the following trigonometric functions. You may use the following formulae:
   \[ \sin(2x) = 2\sin(x)\cos(x), \quad \cos\left(\frac{x}{2}\right) = \pm \sqrt{\frac{1 + \cos(x)}{2}}. \]
   (a) \( \sin(2x) \)
   (b) \( \cos\left(\frac{x}{2}\right) \)

4. Determine if the Intermediate Value Theorem implies that the equation \( 3x^{101} - (x + 1)\cos(x) = 0 \) has a solution in the interval \((-1, 0)\). Explain your answer.

5. Let \( f(x) = \sqrt{x} \). Write down the definition (using limit) of \( f'(x) \). Then find \( f''(x) \) using the definition.

2. Second Midterm

1. Find the limit \( \lim_{h \to 0} \frac{(x + h)^{100} - x^{100}}{h} \). (Hint: The limit represents the derivative of a function.)

2. Find derivatives: \( \frac{dy}{dx} \)
   (a) \( y = \cos(\sin(x)) \).
   (b) \( y = (x^2 + 1)^8(-8x)^{-7} \)

3. Find the equation of the tangent line to the curve
   \( xy^2 - 10xy + 21 = 0 \)
4. The top of a 18 foot ladder, leaning against a vertical wall, is slipping down the wall at the rate of 2 feet per second. How fast is the bottom of the ladder sliding along the ground when the bottom of the ladder is 6 feet away from the base of the wall?

5. Let $f(x) = 5x^2 - 3x + 2$. Explain that $f(x)$ has an absolute maximum and an absolute minimum on the interval $[0, 5]$ and find them.

3. Third Midterm

1. Let $f(x) = (x + 5)^2(x - 6)^3$.
   (a) Find all critical points of $f$.
   (b) Indicate where $f(x)$ is increasing and where $f(x)$ is decreasing.
   (c) Find the $x$-coordinates of all local maxima and minima.

2. A cylinder is inscribed in a right circular cone of height 2 and radius (at the base) equal to 1. What are the dimensions of such a cylinder which has maximum volume?

3. Let $f(x) = \pi \sin(\pi x)$.
   (a) $|f'(x)| \leq k_1$, where $k_1$ is a real number. Find $k_1$.
   (b) By Mean Value Theorem, $|f(b) - f(a)| \leq k_2|b - a|$, where $k_2$ is a real number. Find $k_2$.
   (c) By mean value theorem, we know that there exists at least one $c$ in the interval $(0, 2)$ such that $f'(c)$ is equal to the mean slope. Find all values of $c$.

4. Find $\frac{d}{dx} \int_{t=-5}^{\sqrt{x}} \frac{\cos(t)}{t^{11}} dt$.

5. A particle is moving with acceleration $a(t) = 6t + 2$. Its position at time $t = 0$ is $s(0) = 3$ and its velocity at time $t = 0$ is $v(0) = 2$. What is its position at time $t = 1$?

6. Calculate $\int_{x=0}^{4} (x^2 + 3) \, dx$ by Riemann sum $\lim_{n \to \infty} R_n$. You may need the formula $\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$.
   (a) Find $R_n$ using right end points.
   (b) Find $\lim_{n \to \infty} R_n$. 