The fact that an action of a Lie algebra $\mathfrak{g}$ on a (pre)symplectic manifold $(M, \omega)$ is hamiltonian can be interpreted in terms of the “action Lie algebroid” $\mathfrak{g} \times M$. I will report on work in progress with Christian Blohmann in which we are developing a theory of “Hamiltonian Lie algebroids” which extends this interpretation to general Lie algebroids $\mathcal{A}$. Here, a vector bundle connection on $\mathcal{A}$ replaces the role played by the natural trivialization of $\mathfrak{g} \times M$.

Our work was originally inspired by the problem of understanding in symplectic terms why the initial value constraint manifold in general relativity is coisotropic, but the general theory has turned out to be very rich in itself. Among other things, it extends to Lie algebroids the Atiyah-Bott relation between momentum maps and the Weil model of equivariant cohomology. The problem of determining when the tangent bundle of a symplectic manifold is hamiltonian leads to a simple but unanswered question in pure symplectic topology: does every exact symplectic manifold admits a nowhere vanishing Liouville vector field?