MATH 3210 - SUMMER 2008 - ASSIGNMENT #7

LIMITS, MONOTONIC FUNCTIONS, AND CONTINUOUS FUNCTIONS

(1) Using theorems proven in class find the limits below (and prove them not by definition but appealing to the theorems that we proved).

$$1)\ \lim_{x\to 0}\frac{\tan(x)}{x} \qquad 2)\ \lim_{x\to 0}\frac{\sin(-x)}{x}$$

$$3)\ \lim_{x\to 0}\frac{\sin(6x)}{x} \qquad 4)\ \lim_{x\to\infty}\left(1+\frac{1}{5x}\right)^x$$

$$5)\ \lim_{x\to-\infty}\left(1-\frac{1}{x}\right)^x$$

(2) Check if the following functions are monotonic:

(a) $f:\mathbb{R}\to\mathbb{R},\ f(x)=x^2$

(b) $g:[0,\infty)\to\mathbb{R},\ f(x)=\frac{1}{(x+1)^2}$

(3) What is $\sup_I f$ for the following functions? In each case is it a maximum?

(a) $f:[0,1]\to\mathbb{R},\ f(x)=-8x^2+6x-1$

(b) $f:(0,1)\to\mathbb{R},\ f(x)=2x$

(c) $D:[0,1]\to\mathbb{R},\ D(x)=\begin{cases}1 & \text{if } x\in\mathbb{Q}\\ -1 & \text{if } x\notin\mathbb{Q}\end{cases}$

(d) $f:(0,1)\to\mathbb{R},\ f(x)=\begin{cases}x & \text{if } x\in\mathbb{Q}\\ -x & \text{if } x\notin\mathbb{Q}\end{cases}$

(4) For each function $f(x)$ and point a check if f is continuous at a and if not - find the type of discontinuity:

(a) $f:[0,1]\to\mathbb{R},\ f(x)=\frac{x}{x+1}$ at $a=0$

(b) $f:[-1,1]\to\mathbb{R},\ f(x)=\frac{\sin(x)}{x}$ at $a=0$

(c) $f:(-2,2)\to\mathbb{R},\ f(x)=\sin(\frac{1}{x})$ at $a=0$

(d) $f:\mathbb{R}\to\mathbb{R},\ f(x)=[x]$ at $a=3$

(e) $f : (-2, 2) \to \mathbb{R}$, $f(x) = x \sin(\frac{1}{x})$ at $a = 0$

(f) $f : (-2, 2) \to \mathbb{R}$, $f(x) = x \sin(\frac{1}{x})$ at $a = \frac{1}{2}$

(g) $f : (2, \infty) \to \mathbb{R}$, $f(x) = \frac{x}{[x]}$ at $a = 3$

(h) $f : (2, \infty) \to \mathbb{R}$, $f(x) = \frac{x}{[x]}$ at $a = 3.5$

(5) $f : \mathbb{R} \to \mathbb{R}$, $f(x) = 1$ for all x. $g : \mathbb{R} \to \mathbb{R}$, $g(x) = \begin{cases} x & x \neq 1 \\ 3 & x = 1 \end{cases}$ find $L = \lim_{x\to 5} g(f(x))$, $a = \lim_{x\to 5} f(x)$ and $L' = \lim_{x\to a} g(x)$. Does $L \neq L'$ contradict the theorem proven in class?