MATH 3210 - SUMMER 2008 - ASSIGNMENT #3

ABSOLUTE VALUE AND BOUNDED SETS.

(1) Use induction to prove that for any n real numbers: $a_1, \ldots, a_n$ where $n \geq 2$, $n \in \mathbb{N}$, we have:

$$|a_1 + a_2 + \ldots a_n| \leq |a_1| + |a_2| + \ldots |a_n|$$

This is called the generalized triangle inequality.

(2) (a) Which real x satisfy: $|x-2| + |x+3| > 10$ (Hint: work out the following cases: $x \geq 2$, $-3 \geq x < 2$, and $x < -3$). Draw the set of solutions on the real line.

(b) Solve: $x + 2 < |x^2 - 4|$. Draw the set of solutions on the real line.

(3) Find sup / inf of the following sets, prove your assertions:

(a) $A = (-2, 3)$

(b) $B = \{b | b \in \mathbb{Q} \text{ and } b < 10\}$

(c) $C = \{4 - \frac{1}{n^2} | n \in \mathbb{N}\}$

(d) Find just the inf of $D = \left\{5 + \frac{n}{2n^2+3n+1} \middle| n \in \mathbb{N}\right\}$

(4) For each part, find an example or explain why none exist:

(a) a set A such that $\inf A = 0$ and $\sup A = 15$

(b) a set A such that $\inf A = 15$ and $\sup A = 0$

(c) a set A which is bounded below and not bounded above.

(d) a set A which is bounded above and not bounded below.

(e) a set A which is bounded below and which doesn't have an infimum.

(f) a set A for which $\exists a \in A$ such that $a < 3$ but 3 is not an upper bound for A.

(g) a set A such that:

- $\forall a \in A$: $a \leq 2$
- $\forall \varepsilon > 0$ there is an $a \in A$ such that $a > 2 - \varepsilon$

And $2 \neq \sup A$.

(h) a set A such that:

- $\forall a \in A$: $a \leq 3$
- There is an $a \in A$ such that for all $\varepsilon > 0$: $a > 3 - \varepsilon$

(5) Let A be bounded below. Let $B = \{2 + a | a \in A\}$

(a) If $A = [4, \infty)$ describe B (give a mathematical desciption of B either as a generalized interval or as a set with curly brackets).

(b) If $A = \{1 + \sqrt{n} | n \in \mathbb{N}\}$, describe B.

(c) If $i = \inf A$ prove tht $i + 2 = \inf B$.