MATH 3210 - SUMMER 2008 - ASSIGNMENT #1

Sets

(1) Detrmine wheter each statement is true or false, if it is false, explain why.

(a) $\{2,4,0,6\} = \{2,4,6\}$

(b) $3 \in \{\{3\}\}$

(c) $\{2,3\} \in \{1,2,3\}$

(d) $\{2,3\} \subseteq \{1,2,3\}$

(e) Every even number m can be written as $m = 2k$ for some integer k.

(f) Every odd number n can be written as $n = 2k+1$ for some integer k.

(g) $6 \in \{3k | k \text{ is an integer }\}$

(h) $-6 \in \{3k | k \text{ is an integer }\}$

(i) $0 \in \{3k | k \text{ is an integer }\}$

(j) $6 \in \{5k | k \text{ is an integer }\}$

(k) $\{3k | k \text{ is an integer }\} \subseteq \{6k | k \text{ is an integer }\}$

(l) $\{6k | k \text{ is an integer }\} \subseteq \{3k | k \text{ is an integer }\}$

(2) Prove that $\sqrt[3]{7}$ is irrational. What about $\sqrt[3]{12}$? $\sqrt{4} = 2$ is rational, if we try to follow the same logic as for $\sqrt{2}$ when does the contradiction fail?

(3) (a) Show that for any $r > 0$ there is a natural number n such that $r < \sqrt{2}n$. (hint: Can set $n = [r]$ i.e. r rounded up, but why?)

(b) Prove that for any $0 < s$ there is a natural number n such that $\frac{1}{\sqrt{2}n} < s$

(c) Prove that for any two numbers $x < y$ there are natural numbers n, m such that $x < \frac{m}{\sqrt{2}n} < y$

(d) Is a number of the form $\frac{m}{\sqrt{2}n}$ (where n, m are natural) rational or irrational - prove your claim.