MATH 3210 - SUMMER 2008 - PRACTICE MIDTERM

You have an hour and a half to complete this test. The maximum grade is 100.

question	**grade**	**out of**
1		33
2		33
3a		16
3b		8
3c		6
3d		4
total		100

Student Number: ______________________________

(1) (33 pts) Using the definition of a convergent sequence prove the following theorem (Do not appeal to any theorem):

If $\{a_n\}_{n=1}^{\infty}$ converges to a and $a > 0$ then there is an $N_1 \in \mathbb{N}$ such that for all $n > N_1$: $a_n > 0$.

(2) (33 pts) Consider the following sequence defined inductively:

$$a_1 = 1$$
$$a_{n+1} = \sqrt[3]{(a_n)^2 + 2a_n}$$

Prove that $\{a_n\}_{n=1}^{\infty}$ converges and find its limit.

(3) (34 pts) For each of the following statements, determine if they are true or false. If they are true, prove them. You are allowed and encouraged to appeal to the theorems proven in class (without proof) as long as you quote them in full. If the statement is false find a counter example.

(a) (16 pts) True/False:

If $\lim_{n\to\infty} a_n = 0$ and $\{b_n\}_{n=1}^{\infty}$ satisfies:

$$-\frac{1}{n^2} \leq b_n \leq a_n + \frac{3}{n}$$

for all $n \in \mathbb{N}$. Then $\{b_n\}_{n=1}^{\infty}$ converges.

(b) (8 pts) True/False:

Suppose $\{a_n\}_{n=1}^{\infty}$, $\{b_n\}_{n=1}^{\infty}$ are sequences which satisfy the following properties:

(i) $\lim_{n\to\infty} a_n = 0$, $\lim_{n\to\infty} b_n = 0$

(ii) $b_n \neq 0$ for all $n \in \mathbb{N}$

then $\lim_{n\to\infty} (a_n)^{b_n} = 0$

(c) (6 pts) True/False:

If $\{a_n\}_{n=1}^{\infty}$ diverges then $\{a_n\}_{n=1}^{\infty}$ is not bounded.

(d) (4 pts) True/False:

There is a strictly monotonically increasing sequence $\{a_n\}_{n=1}^{\infty}$ of natural numbers such that the sequence $b_n = \cos(a_n)$ converges.