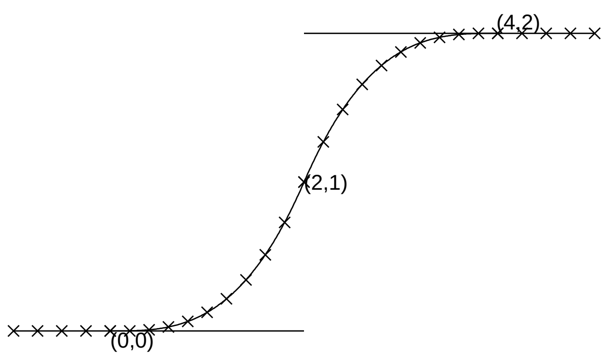
Math 5610 **Homework 8** **Due Dec. 15, 2005**
Grady Wright **8:00am**

This homework can only be used to replace a previous homework assignment.

1. The Utah Transit Authority (UTA) was so impressed with your design of the switching path for the two parallel lines of TRAX (refer to homework 5), that they have requested your services on a new project. This new project is exactly the same as the previous project except now the slope at the midpoint of the switching path has been prescribed (as opposed to being a free parameter). The exact requirements for the switching path are that it pass through the points $(0,0)$, $(2,1)$, and $(4,2)$. Furthermore, the path should be tangent to the line $y = 0$ at $(0,0)$, tangent to the line $y = 2$ at $(4,2)$, *and* have a slope of $3/2$ at $x = 2$ (see the figure below).



   a. Since you are now given 6 constraints for the path, you may decide to use a 5th degree (or lower) polynomial to describe the path. Explain why this is a bad idea.

      Hint: Compute the polynomial of degree ≤ 5 that satisfies the constraints and then plot it for $x \in [0,4]$. Does the path described by the polynomial seem like a good idea?

   b. Now try using a piecewise polynomial interpolant to design the path. Does the resulting path look like a plausible solution?

      Hint: How many intervals are there for the problem? How many constraints are there for each interval? What degree polynomial should be used in each interval? Plot the path from the piecewise polynomial interpolant you designed.

2. A natural cubic spline s for a function f is defined on $[1,4]$ by

$$s(x) = \begin{cases} s_0(x) = b_0(x-1) + d_0(x-1)^3 & 1 \leq x < 3, \\ s_1(x) = 1 + b_1(x-3) - \frac{3}{4}(x-3)^2 + d_1(x-3)^3 & 3 \leq x \leq 4. \end{cases}$$

   a. If $s(x)$ interpolates the data $(1,0), (3,1), (4,0)$, find b_0, d_0, b_1, and d_1.

   b. Find the second degree global polynomial interpolant $p_2(x)$ to the same data from part a. Verify that $\int_1^4 [s''(x)]^2 dx$ is less than $\int_1^4 [p''(x)]^2 dx$.

3. Assuming you have computed the LU decomposition of an $n \times n$ nonsingular matrix A using Gaussian elimination with partial pivoting (i.e. $A = P^T LU$). Explain (in pseudocode or in steps) how you would efficiently solve the following problems:

   a. Solve the linear system $A^k x = b$, where k is a positive integer $< n$.

   b. Solve the matrix equation $AX = B$, where B is $n \times m$.

   Hint: None of your solutions should involve another $P^T LU$ decomposition.

4. *Plane of best fit*

   a. Given experimental data $(x_j, y_j, f(x_j, y_j))$, $j = 1, 2, \ldots, n$, determine the plane $p(x, y) = a + bx + cy$ that best fits this data in the least-squares sense, i.e. deterine the values of the coefficients a,b, and c such that the l_2 norm $\sqrt{\sum_{j=1}^{n}[f(x_j, y_j) - p(x_j, y_j)]^2}$ is minimized.

   b. Employees at a leading shoe store were given a psychological exam to measure their intelligence (a score on a scale of 50-low intelligence to 150-high intelligence) and extroversion (a score on a scale of 15-low extroversion to 30-high extroversion). The company then compared these two numbers with the average amount of sales each employee generated per week. The data is given in the table below:

Sales Person	Intelligence	Extroversion	Sales/Week
1	89	21	2625
2	93	24	2700
3	91	21	3100
4	122	23	3150
5	115	27	3175
6	100	18	3100
7	98	19	2700
8	105	16	2475
9	112	23	3625
10	109	28	3525

   Determine the plane of best fit to this data and use it to predict the average weekly sales of an employee with an intelligence score of 150 and an extroversion score of 18.

5. Bradie p. 532, problems 17 and 18. You may use any numerical quadrature method of your choice (even those built into MATLAB or *Maple*) to evaluate the integral. However, you must show the steps you used to avoid the singularities of the integrand.