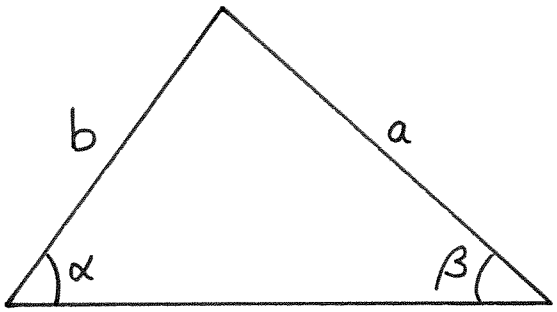
Law of Sines

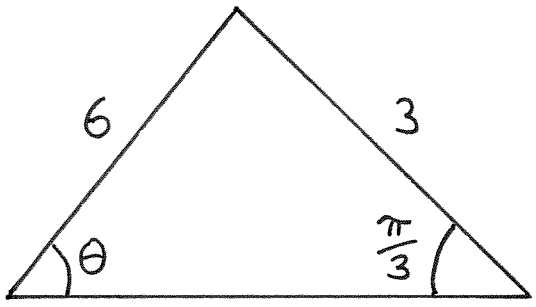
Suppose we have a triangle with two of its angles, α and β, identified. Suppose further that the length of the side of the triangle that is opposite the angle α is a, and that the length of the side opposite β equals b.



Then the *law of sines* is the formula

$$\frac{\sin(\alpha)}{a} = \frac{\sin(\beta)}{b}$$

Problem. Find $\sin(\theta)$ if θ is the angle shown in the triangle below.



Solution. The law of sines tells us that

$$\frac{\sin(\theta)}{3} = \frac{\sin\left(\frac{\pi}{3}\right)}{6}$$

Multiplying both sides of the equation by 3 gives us

$$\sin(\theta) = \frac{\sin\left(\frac{\pi}{3}\right)}{2}$$

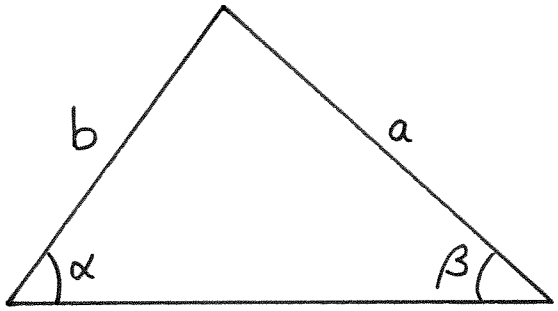
and we know that $\sin\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2}$ so

$$\sin(\theta) = \frac{\frac{\sqrt{3}}{2}}{2} = \frac{\sqrt{3}}{4}$$

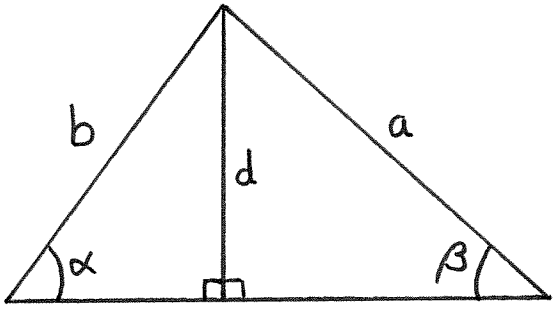
* * * * * * * * * * * * *

Why the law of sines is true

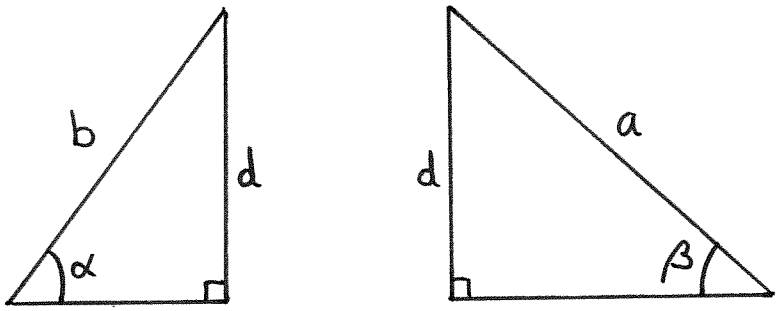
If we have the triangle



then we can draw a line from the top of the triangle to the base of the triangle in a way that creates a right angle. We'll say that this new line has length d.



Now we have divided our original triangle into two right triangles. One on the left and one on the right.



The right triangle on the left can be used to find $\sin(\alpha)$. It's the length of the opposite side divided by the length of the hypotenuse. That is, $\sin(\alpha) = \frac{d}{b}$. Thus, $b\sin(\alpha) = d$.

Similarly, $\sin(\beta)$ is the length of the opposite side divided by the length of the hypotenuse, which is to say that $\sin(\beta) = \frac{d}{a}$. Thus, $a\sin(\beta) = d$.

Putting together the two equations $b\sin(\alpha) = d$ and $a\sin(\beta) = d$ gives us that

$$b\sin(\alpha) = d = a\sin(\beta)$$

which we can write more simply as

$$b\sin(\alpha) = a\sin(\beta)$$

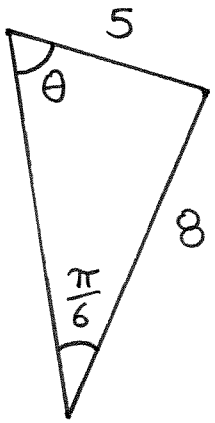
Now divide both sides by ab to see that

$$\frac{\sin(\alpha)}{a} = \frac{\sin(\beta)}{b}$$

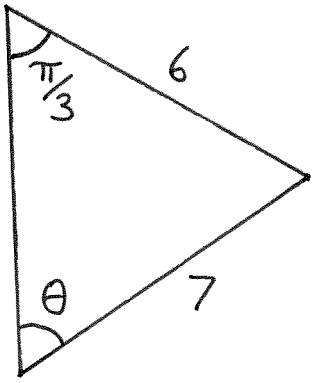
Exercises

Find $\sin(\theta)$. You can consult the chart on page 227 in the chapter "Sine and Cosine" to find the values of sine that you need to complete these problems.

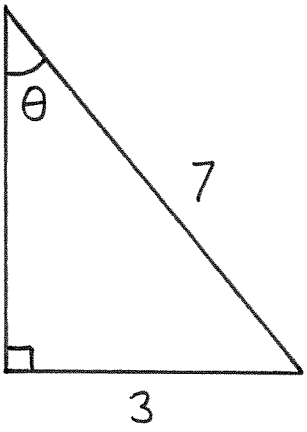
1.)



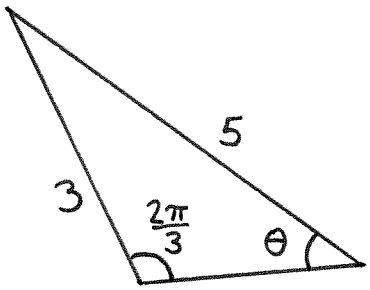
2.)



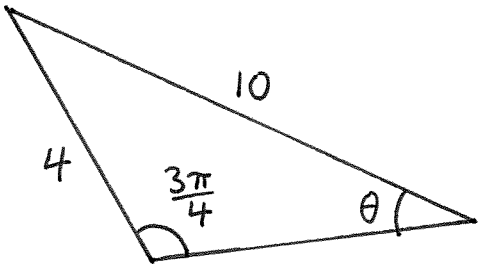
3.)



4.)



5.)



6.)