Equations in One Variable I

An *equation in one variable* is an equation of the form $f(x) = g(x)$ where $f(x)$ and $g(x)$ are functions.

Examples.

- $x^2 - 3x + 2 = \log_e(x)$ is an equation in one variable. Both $x^2 - 3x + 2$ and $\log_e(x)$ are functions.

- $x - 3 = 2$ is an equation in one variable. Both $x - 3$ and 2 are functions. The latter is a constant function.

- $\sqrt{x} = 15x - 4$ is an equation in one variable.

- $\frac{x^2+7}{x-1} + 27x - 3 = e^{x^3-18}$ is an equation in one variable.

All of the examples above are *equations*, because they are mathematical formulas that use an equal sign. The are *in one variable*, because there is a single variable used in each equation. In all of the examples above, x is the only variable. We'll continue to use x as the only variable for a while, but keep in mind that it's just a variable, and that we could use any variable instead. For example, there really isn't any difference between the equation $x - 3 = 2$ and the equation $w - 3 = 2$. The only difference is the name that we gave the variable, x or w, but otherwise the equations are the same.

Domain of an equation

If $f(x) = g(x)$ is an equation in one variable, then the *implied domain* of the equation is the set of all real numbers that are in both the implied domain of f and the implied domain of g.

Examples.

- The implied domain of the equation $x^3 - 2x + 1 = 3x - 9$ is $\mathbb{R}$. That's because $x^3 - 2x + 1$ and $3x - 9$ are each polynomials, so they each have $\mathbb{R}$ as their implied domains.

- The equation $\sqrt{x} = 4$ has an implied domain of $[0, \infty)$. That's because $\sqrt{x}$ has an implied domain of $[0, \infty)$, while the constant function 4 has an implied domain of $\mathbb{R}$. Therefore, the implied domain of $\sqrt{x} = 4$ is the set of all real numbers that are in the set $[0, \infty)$ and in the set $\mathbb{R}$, or in other words, the implied domain of $\sqrt{x} = 4$ is the set $[0, \infty)$.

- The implied domain of the equation $\log_e(x) + \log_e(x-2) = 1$ is $(2, \infty)$. The implied domain of the constant function 1 is $\mathbb{R}$. The implied domain of $\log_e(x) + \log_e(x-2)$ is a little more tricky. We can only take the logarithm of a positive number. That means that we want $x > 0$ and that we want $x - 2 > 0$, or equivalently, that $x > 2$. Of course asking that $x > 0$ and $x > 2$ is the same as just asking that $x > 2$, so the implied domain for the function $\log_e(x) + \log_e(x-2)$ is $(2, \infty)$.

- The implied domain of the equation $\sqrt{x} = \frac{3x}{x-5}$ is $[0, \infty) - \{5\}$. We can only take the square-root of a number if it is greater than or equal to 0, so $x \geq 0$. We can never divide by 0, so $x - 5 \neq 0$, or equivalently $x \neq 5$. The set of all real numbers x with $x \geq 0$ and $x \neq 5$ is the set $[0, \infty) - \{5\}$.

- $[3, 5)$ is the implied domain of the equation $\log_e(5 - x) = \sqrt{x-3}$. We can only take the logarithm of a number if it is positive, so $5 - x > 0$, or equivalently, $5 > x$. We can only take the square-root of a number if it is greater than or equal to 0, so $x - 3 \geq 0$, or equivalently, $x \geq 3$. The set of all real numbers x with $5 > x$ and $x \geq 3$ is exactly $[3, 5)$.

Sometimes, you will be given an explicit domain for an equation. For example, you might be asked to examine the equation $x^2 - 3x + 1 = e^x$ for $x \in [0, 10]$. This means that the domain of the equation is the restricted domain $[0, 10]$. The domain is restricted, because the implied domain of the equation $x^2 - 3x + 1 = e^x$ would have been $\mathbb{R}$ if we hadn't been told otherwise, but we have been told otherwise. We are told here that the domain of the equation is just the set $[0, 10]$.

Solutions of equations

If someone writes an equation such as $3x - 2 = 4$, they are not saying that $3x - 2$ is the same function as the constant function 4. They are clearly not the same function. One is a constant function and the other one isn't. Rather, they are asking you to find the set of all numbers that can be substituted in for x to make the equation true. For example, we can substitute 2 in the place of x and we'd be left with the equation $3(2) - 2 = 4$, which is true, as you can check. The equation usually won't be true for every number that you substitute for x as we can see by substituting the number 0 in the place of x.

That would leave us with $3(0) - 2 = 4$, which is clearly false. The numbers that can be substituted for x to make an equation true are called *solutions* of the equation.

If $f(x) = g(x)$ is an equation in one variable, and if the equation has the set D as its domain, then the set of solutions of $f(x) = g(x)$ is the set

$$S = \{ \alpha \in D \mid f(\alpha) = g(\alpha) \}$$

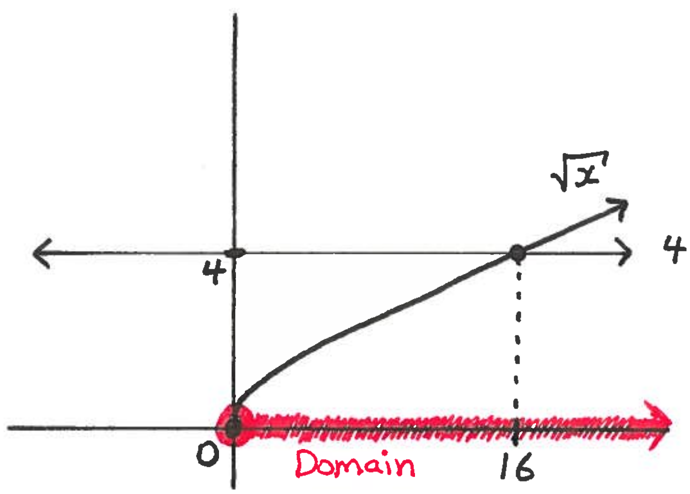
Returning to the example above, if S is the set of solutions of the equation $3x - 2 = 4$, then $2 \in S$, but $0 \notin S$. We'd speak this as 2 is a solution of $3x - 2 = 4$, but that 0 is not a solution of $3x - 2 = 4$.

Examples.

• Suppose that S is the set of solutions of the equation $\sqrt{x} = 4$. We saw above that this equation has a domain of $[0, \infty)$, so

$$S = \{ \alpha \in [0, \infty) \mid \sqrt{\alpha} = 4 \}$$

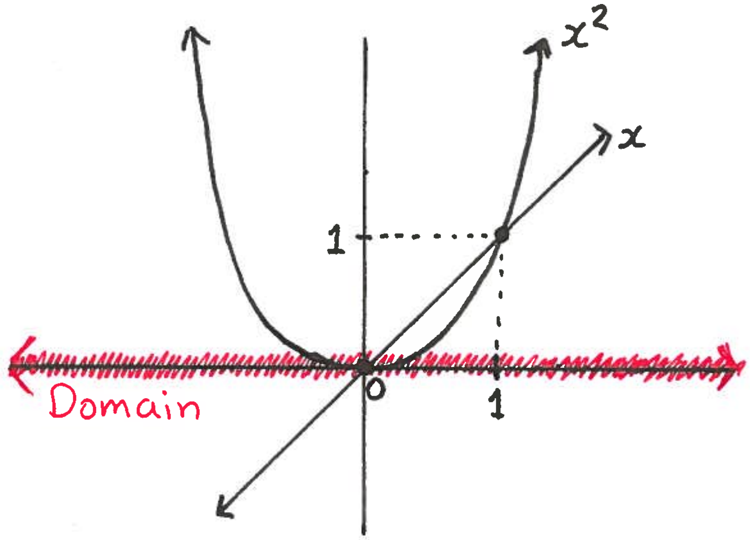
We see that $16 \in S$, since $16 \in [0, \infty)$ and $\sqrt{16} = 4$. However, $-10 \notin S$ since $-10 \notin [0, \infty)$, and $25 \notin S$, since $\sqrt{25} \neq 4$.



• Now suppose that S is the set of the solutions for the equation $x^2 = x$. Both x^2 and x are polynomials, so the domain of the equation is $\mathbb{R}$ and

$$S = \{ \alpha \in \mathbb{R} \mid \alpha^2 = \alpha \}$$

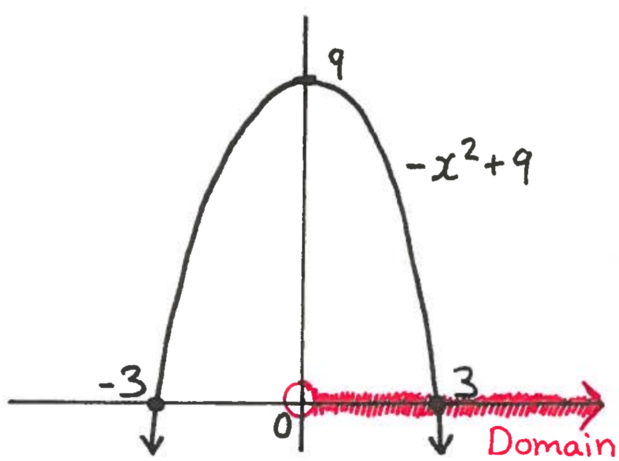
Then $0, 1 \in S$ because 0 and 1 are both real numbers and $0^2 = 0$ and $1^2 = 1$. However, $3 \notin S$ because $3^2 \neq 3$.



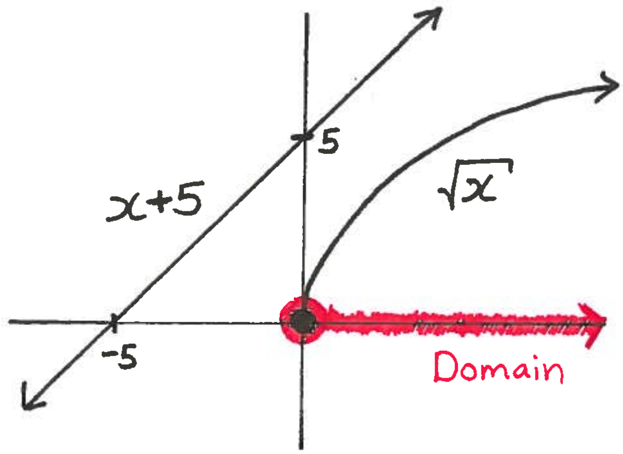
• Let's look at an equation with a restricted domain. Let's look at the equation $-x^2 + 9 = 0$ for $x \in (0, \infty)$. We are told here that the domain is $(0, \infty)$, which means that we are only interested in finding solutions to the equation if those solutions are positive numbers. That is, if S is the set of solutions, then

$$S = \{ \alpha \in (0, \infty) \mid -\alpha^2 + 9 = 0 \}$$

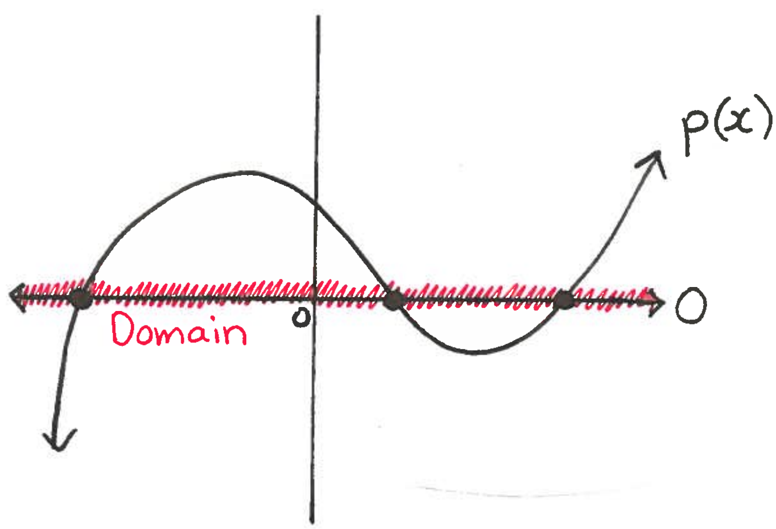
Notice that S is the set of all positive numbers that are also roots of the quadratic polynomial $-x^2 + 9 = 0$. We know how to find roots of quadratic polynomials, and the roots of $-x^2 + 9$ are -3 and 3. Of these two roots, only 3 is in the domain of the equation. That is, only 3 is positive. Therefore, $S = \{3\}$.



• The equation $\sqrt{x} = x + 5$ has an implied domain of $[0, \infty)$, since we can't take the square-root of a negative number. We can see below that the graphs of $\sqrt{x}$ and $x + 5$ do not intersect. That means there isn't a number α with $\sqrt{\alpha} = \alpha + 5$. That is to say, there aren't any solutions of the equation $\sqrt{x} = x + 5$. The set of solutions is $\emptyset$.



• An important general example is if $p(x)$ is a polynomial and our equation is $p(x) = 0$. Then the set of solutions for this equation is exactly the set of roots of the polynomial $p(x)$. We know how to find roots of any linear and quadratic polynomial, but it is extremely difficult to find roots of most polynomials. A polynomial equation $p(x) = 0$ is about as nice of an equation as we can hope to have, and even finding its solutions can be a nearly impossible task.



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In the remainder of this chapter, we'll cover a few basic rules of algebra that can be used to help us find solutions of equations. We'll learn when equations are equivalent — either by addition, multiplication, or invertible functions — and we'll see that equivalent equations have the same solutions. That will allow us to find solutions of equations by creating a sequence of equivalent equations that terminates in an equation that's easy for us to solve.

Equations equivalent by addition

In the definition below, D is a set of real numbers. That is, $D \subseteq \mathbb{R}$.

The equation

$$f(x) + h(x) = g(x) \quad \text{with domain } D$$

is *equivalent* to the equation

$$f(x) = g(x) - h(x) \quad \text{with domain } D$$

Examples.

• The equation $x^2 + 3x = e^x$ with domain $\mathbb{R}$ is equivalent to the equation $x^2 = e^x - 3x$ with domain $\mathbb{R}$. Here we are just using the rule above with $f(x) = x^2$, $h(x) = 3x$, and $g(x) = e^x$.

• The domains of two equivalent equations must be the same. The equation $x^2 + 3x = e^x$ with the restricted domain $(0, \infty)$ is equivalent to the equation $x^2 = e^x - 3x$ with the restricted domain $(0, \infty)$.

• To repeat, the domains of two equivalent equations are equal. The equation $x^2 + 3x = e^x$ with domain $[-3, 10]$ is equivalent to the equation $x^2 = e^x - 3x$ with domain $[-3, 10]$. Compare this example to the previous two examples. Equivalent equations have the same domain.

• $x^2 - \log_e(x) = 0$ is equivalent to $x^2 = \log_e(x)$. To see this, just subtract $-\log_e(x)$ (or add $\log_e(x)$) to both sides of the equation $x^2 - \log_e(x) = 0$. Both equations in this example have an implied domain of $(0, \infty)$, because we can only take the logarithm of a positive number.

• Whenever you add or subtract a function from both sides of an equation, you'll obtain an equivalent equation. Thus, $7x^3 - 5 = x^2 + 2x$ is equivalent to $7x^3 - x^2 - 2x - 5 = 0$. Just subtract $x^2 + 2x$ from both sides of $7x^3 - 5 = x^2 + 2x$.

• $\sqrt{3x} = 2x^2$ is equivalent to $\sqrt{3x} + x - 2 = 2x^2 + x - 2$. Just add $x - 2$ to both sides of $\sqrt{3x} = 2x^2$.

The importance of equivalent equations is that they have the same sets of solutions.

Equivalent equations
have the same sets of solutions.

Examples.

• The equations $x^2 = -x + 5$ and $x^2 + x - 5 = 0$ are equivalent. To get the second equation from the first, just add the function $x - 5$.

Because these two equations are equivalent, they have the same set of solutions. We know how to find the solutions of $x^2 + x - 5 = 0$. These are just the roots of the quadratic polynomial $x^2 + x - 5$, which are $\frac{1}{2}(-1 - \sqrt{21})$ and $\frac{1}{2}(-1 + \sqrt{21})$. In other words, the set of solutions of the equation $x^2 + x - 5 = 0$ is the set $\{\frac{1}{2}(-1 - \sqrt{21}), \frac{1}{2}(-1 + \sqrt{21})\}$. Therefore, the set of solutions of $x^2 = -x + 5$ is also $\{\frac{1}{2}(-1 - \sqrt{21}), \frac{1}{2}(-1 + \sqrt{21})\}$.

• To find the set of solutions of $x^2 + x - 4 = x^2$, just subtract x^2 from both sides of the equation. We'll be left with the equivalent equation $x - 4 = 0$. The set of solutions of this latter equation is $\{4\}$. Because the two equations in this example are equivalent, the set of solutions of $x^2 + x - 4 = x^2$ is also $\{4\}$.

• To find the solutions of $\sqrt{x} - x = 5$, we can add the function x to both sides of the equation and we'd be left with the equivalent equation $\sqrt{x} = x + 5$. We saw earlier in this chapter that $\sqrt{x} = x + 5$ has no solutions. Therefore, $\sqrt{x} - x = 5$ has no solutions.

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Equations equivalent by multiplication

We say that the number $\alpha \in \mathbb{R}$ is a *zero* of the function $h(x)$ if $h(\alpha) = 0$.

Examples.

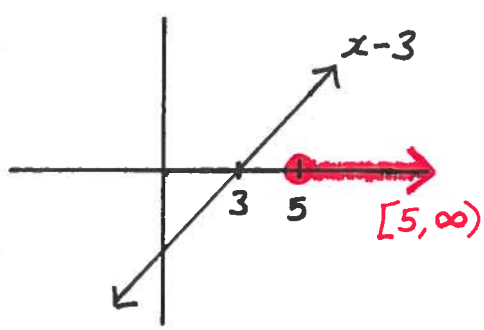
- 2 is a zero of $h(x) = x^2 - 4$ because $h(2) = 2^2 - 4 = 0$. The number -2 is also a zero of $h(x) = x^2 - 4$.
- 3 is a zero of $h(x) = x - 3$ since $h(3) = 3 - 3 = 0$.
- In the previous two examples, $h(x)$ was a polynomial. If $h(x)$ is a polynomial then we usually call a zero of $h(x)$ a *root* of $h(x)$ instead. It's just a different name for the same thing.
- 5 is a zero of $\sqrt{x-5}$ because $\sqrt{5-5} = \sqrt{0} = 0$.
- 1 is a zero of $\log_e(x)$.
- 4 is *not* a zero of $2x - 5$ because $2(4) - 5 = 8 - 5 = 3 \neq 0$.

We say that $h(x)$ has *no zeros in* a set $D \subseteq \mathbb{R}$ if none of the numbers in D is a zero for $h(x)$.

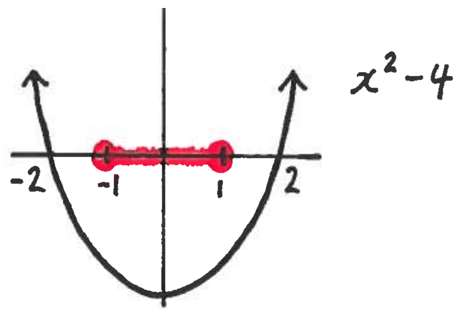
Examples.

- The linear polynomial $h(x) = x - 3$ has no zeros in $[5, \infty)$. That's because if $\alpha \in [5, \infty)$, then $\alpha \geq 5$. Therefore, $h(\alpha) = \alpha - 3 \geq 5 - 3 = 2$, and if $h(\alpha) \geq 2$, then $h(\alpha) \neq 0$. That is, α is not a zero of $h(x)$ if $\alpha \in [5, \infty)$.

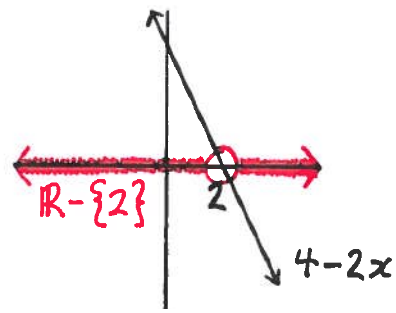
The function $h(x) = x - 3$ does have a zero. It's 3. But $h(x)$ has no zeros in $[5, \infty)$.



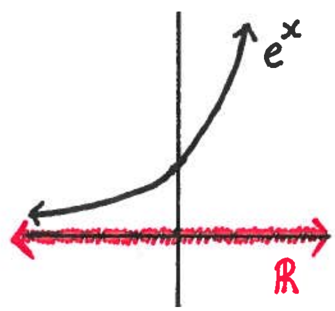
• The function $h(x) = x^2 - 4$ has two zeros, 2 and -2. Neither of these zeros are in the set $[-1, 1]$, so $h(x) = x^2 - 4$ has no zeros in $[-1, 1]$.



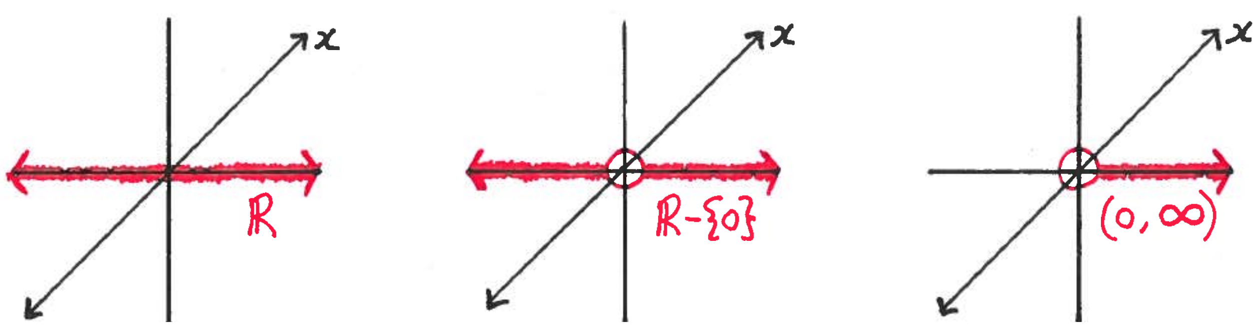
• $4 - 2x$ has exactly one zero, the number 2. Because 2 is the only zero, $4 - 2x$ has no zeros in $\mathbb{R} - \{2\}$.



• e^x has no zeros. It has no zeros in $\mathbb{R}$.



• x is a function. It's the identity function. It only has one zero, the number 0 itself. Thus, x has a zero in $\mathbb{R}$, but it has no zeros in $\mathbb{R} - \{0\}$. Neither does it have zeros in $(0, \infty)$, for example.



In the definition below, D is a subset of real numbers.

The equation

$$h(x)f(x) = g(x) \quad \text{with domain } D$$

is *equivalent* to the equation

$$f(x) = \tfrac{g(x)}{h(x)} \quad \text{with domain } D$$

as long as $h(x)$ has no zeros in D.

The definition above tells us that we can multiply or divide both sides of an equation by a function—as long as the function has no zeros in the domain—and obtain an equivalent equation.

Examples.

• If $h(x)$ is a constant function, and not the constant 0, then it has no zeros. Therefore, we can always divide both sides of an equation by $h(x)$ to obtain an equivalent equation. Thus, $3x = x^4 + 1$ is equivalent to $x = \frac{x^4+1}{3}$. The equation $2\sqrt{x} = 8$ is equivalent to $\sqrt{x} = 4$.

• If we multiply both sides of an equation by a constant that's not zero then we'll also obtain an equivalent equation. $\frac{x}{4} = \log_5(x)$ is equivalent to $x = 4\log_5(x)$. $\frac{1}{3}\sqrt{x} = 4$ is equivalent to $\sqrt{x} = 12$.

• The equation $x\log_e(x) = x$ has a domain of $(0, \infty)$. Because the function x has no zeros in $(0, \infty)$, we can divide both sides of the equation $x\log_e(x) = x$ by x to obtain the equivalent equation $\log_e(x) = 1$.

• This is an example of something that can't be done. If you have the equation $x^2 = x$, then you might be inclined to divide by x, similar to the previous example. We can't do that here though. The implied domain of the equation $x^2 = x$ is $\mathbb{R}$, and x has a zero in $\mathbb{R}$. It has the number 0 as its zero.

The two equations $x^2 = x$ and $x = 1$ (which is what we would get by dividing $x^2 = x$ by x) are very different equations. The former has two solutions, 0 and 1, while the latter has a single solution, 1.

Equivalent equations
have the same solutions.

Examples.

• The equation $2\sqrt{x} = 8$ is equivalent by multiplication to $\sqrt{x} = 4$. We know the set of solutions of $\sqrt{x} = 4$ is $\{2\}$, so $2\sqrt{x} = 8$ also has 2 as its only solution.

• The equation $x \log_e(x) = x$ is equivalent to $\log_e(x) = 1$. The only solution of $\log_e(x) = 1$ is the number e. Therefore, e is the only solution of the equation $x \log_e(x) = x$.

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Equations equivalent by invertible function

Once again, in the definition below $D \subseteq \mathbb{R}$.

The equation

$$h(f(x)) = g(x) \quad \text{with domain } D$$

is *equivalent* to the equation

$$f(x) = h^{-1}(g(x)) \quad \text{with domain } D$$

The above definition says that we can erase a function, h, on one side of an equation if we apply its inverse, h^{-1} to the other side of the equation. Both equations have to have the same domain. Equivalent equations always have the same domain.

Examples.

• $\log_e(x) = 5$ has an implied domain of $(0, \infty)$, because we can only take the logarithm of a positive number. We could erase logarithm base e on the left side of $\log_e(x) = 5$ by applying its inverse, exponential base e to the right side. We'd be left with the equation $x = e^5$ with the same domain that we started with, the set $(0, \infty)$. These two equations, $\log_e(x) = 5$ and $x = e^5$, with the same domain, $(0, \infty)$, are equivalent.

Equivalent equations always have the same domain.

• The functions $\sqrt[3]{x}$ and x^3 are inverse functions. Therefore, $\sqrt[3]{x-2} = 7$ is equivalent to $x - 2 = 7^3$. (Both equations have a domain of $\mathbb{R}$ because we can cube or cube-root any number.) We erased the cube root from the left side of $\sqrt[3]{x-2} = 7$ by applying its inverse to the right side.

Again, what's important about equivalent equations is the following fact:

Equivalent equations
have the same solutions

Example.

• We'll continue from the previous example. If we have the equation $\sqrt[3]{x-2} = 7$, then we know it's equivalent to the equation $x - 2 = 7^3$. Now $7^3 = 343$, so the previous equation is $x - 2 = 343$. The only solution of this equation is $x = 343 + 2 = 345$.

Because $\sqrt[3]{x-2} = 7$ and $x - 2 = 343$ are equivalent, they have the same solutions. Therefore, 345 is the only solution of $\sqrt[3]{x-2} = 7$.

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Finding solutions of equations

To find the solutions of an equation $f(x) = g(x)$, first write down its domain D. Next, find a sequence of equivalent equations, either equivalent by addition, or by multiplication, or by invertible function. The goal is to have the last equation in this sequence have a set of solutions that is easy to find. It might be really easy, such as in the equation $x = 2$, or it might be somewhat easy to find, as in the equation $x^2+2x-5 = 0$, where you can apply the quadratic formula to the quadratic polynomial x^2+2x-5. Whatever easy equation you are left with, find its set of solutions S, but remember to only include in the set S those solutions that are in the domain D of your original equation. Because equivalent equations have the same set of solutions, S will be the set of solutions of your original equation $f(x) = g(x)$.

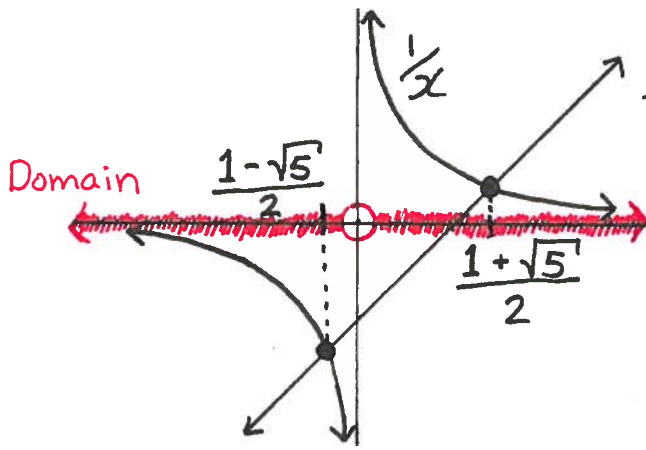
Examples.

• Let's find the solutions of the equation $e^{x^2+x-6} = 1$ where the domain of the equation is the set $D = (0, \infty)$. That means we are only interested in those solution of $e^{x^2+x-6} = 1$ that are positive.

Using $\log_e$, the inverse function of exponential base e, the equation $e^{x^2+x-6} = 1$ is equivalent to $x^2+x-6 = \log_e(1)$, which is the equation $x^2+x-6 = 0$. To find the solutions of this quadratic equation, we use the quadratic formula to find the roots of the quadratic polynomial $x^2 + x - 6$. In doing so, we would find that the solutions of $x^2 + x - 6 = 0$ are -3 and 2. However, the domain of our original equation is $(0, \infty)$, and $-3 \notin (0, \infty)$. Therefore, the only relevant solution for us is 2, because $2 \in (0, \infty)$. That means that $S = \{2\}$ is the set of solutions for our original equation $e^{x^2+x-6} = 1$ with a domain of $(0, \infty)$.

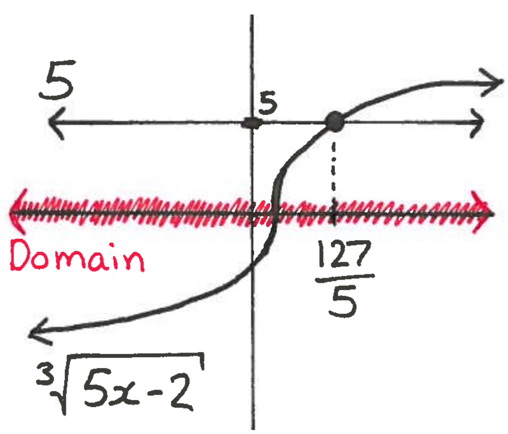
• Let's find the solutions of $x - 1 = \frac{1}{x}$. If we aren't told what the domain of the equation is, then we have to find its implied domain. For this equation, the implied domain is $\mathbb{R} - \{0\}$, because we can't divide by 0. That means that we should only search for solutions of the equation that aren't zero.

We can multiply by the function x to obtain the equivalent equation $x^2 - x = 1$. We can add -1 to obtain the equivalent equation $x^2 - x - 1 = 0$. The quadratic formula tells us that the solutions to this quadratic equation are $\frac{1}{2}(1 - \sqrt{5})$ and $\frac{1}{2}(1 + \sqrt{5})$. Neither of these numbers are 0, so they are both in the domain of our original equation, and they are both solutions of our original equation, $x - 1 = \frac{1}{x}$. They are the only solutions of $x - 1 = \frac{1}{x}$.

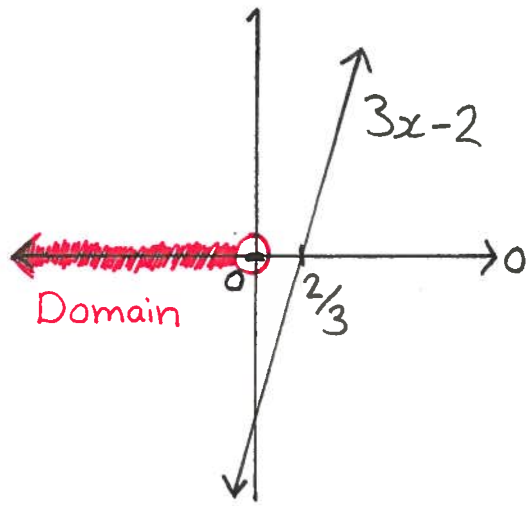


• The equation $\sqrt[3]{5x - 2} = 5$ has an implied domain of $\mathbb{R}$. We can obtain the equivalent equation $5x - 2 = 5^3 = 125$ by erasing the cube-root from the left and applying the cube to the right. Now we can add 2 to obtain $5x = 127$, and divide by 5 to have $x = \frac{127}{5}$.

Of course $\frac{127}{5}$ is in the domain of our original function, since $\frac{127}{5}$ is a real number. Therefore, the only solution of $\sqrt[3]{5x - 2} = 5$ is $\frac{127}{5}$.



• The equation $3x - 2 = 0$, has a solution. It's $\frac{2}{3}$. However, if we are told to find the solutions of $3x - 2 = 0$ with a domain of $(-\infty, 0)$, then there are no solutions. The set of solutions is $\emptyset$. That's because $\frac{2}{3}$ isn't a negative number.



Exercises

For #1-13, match the numbered equations on the left to their lettered implied domains on the right.

1.) $\log_e(x) - \log_e(x) = 4$

2.) $x^2 - 45 = x^7 - 3x - 6$

3.) $e^{-7x^2-3x} = x^3$

4.) $\frac{\sqrt{x}}{\sqrt{x}} = 3x - 2$

5.) $\log_e(x-5) = 23x^9$

6.) $\sqrt[171]{3x-5} = 2x+3$

7.) $\frac{2x^2-7x+3}{x-5} = 3x+4$

8.) $\log_e(x+7) = \sqrt{x-2}$

9.) $\sqrt[3]{x-7} = 3x+4$

10.) $x^5 + \frac{7x+3}{x^2-4} = \sqrt{x}$

11.) $\sqrt[20]{x-2} = 3x+6$

12.) $3x - 4x^6 = x^3 - 13x + 56$

13.) $\sqrt{x} = 3x^2 - 5$

A.) $\mathbb{R}$

B.) $(0, \infty)$

C.) $[0, \infty)$

D.) $[2, \infty)$

E.) $[0, \infty) - \{2\}$

F.) $(5, \infty)$

G.) $\mathbb{R} - \{5\}$

For #14-19, match the numbered equations on the left to their lettered sets of solutions on the right.

14.) $4x - 3 = 0$

15.) $2x - 6 = 0$

16.) $x + 4 = 0$

17.) $x^2 - 2x + 3 = 0$

18.) $2x^2 - 12x + 18 = 0$

19.) $-x^2 - 2x + 7 = 0$

A.) $\{ \frac{3}{4} \}$

B.) $\{ -1 - 2\sqrt{2}, \ -1 + 2\sqrt{2} \}$

C.) $\emptyset$

D.) $\{-4\}$

E.) $\{3\}$

For #20-24, match the numbered equations on the left to the lettered equations on the right that they are equivalent to by addition.

20.) $x^2 + x = 3$

21.) $\log_e(x) + 2x - 3 = 0$

22.) $5 - \sqrt{x} = 3x + 2$

23.) $3x^2 - 4x + 5 = x^2 + 3x + 2$

24.) $-x^2 - 4 = 3x - 7$

A.) $5 = 3x + \sqrt{x} + 2$

B.) $2x^2 - 7x + 3 = 0$

C.) $x^2 + x - 3 = 0$

D.) $-x^2 - 3x + 3 = 0$

E.) $2x - 3 = -\log_e(x)$

If you ever have an equation $p(x) = q(x)$ where both $p(x)$ and $q(x)$ are polynomials, it's almost always best to begin by writing the equivalent equation $p(x) - q(x) = 0$, and then to search for solutions of $p(x) - q(x) = 0$. Use this method to find the solutions of the equations in problems #25-28.

25.) $x^2 - 3x + 2 = 3x^2 + x - 4$

26.) $x^3 + 3x - 2 = x^3 + x^2 + x + 1$

27.) $x^2 - 2x + 3 = x^2 - x + 4$

28.) $x - 2 = 3x^2 - 5x + 1$

For #29-35, match the numbered functions on the left with their lettered set of zeros on the right.

29.) $x - 7$	A.) $\{-1, 1\}$
30.) $x + 5$	B.) $\{0\}$
31.) $x^2 - 1$	C.) $\{-5\}$
32.) $\log_e(x)$	D.) $\{1\}$
33.) $\sqrt{x}$	E.) $\emptyset$
34.) $x^2 + 1$	F.) $\{7\}$
35.) x	

For each equation given in #36-40, decide whether dividing by the function x on both sides of the equation would result in an equivalent equation. For each of these, you'll want to check whether x has zeros in the implied domain of the given equation.

36.) $3x^2 - 5 = 2x + 8$

37.) $\log_e(x) = 3x - 5$

38.) $\frac{1}{x-7} = 3x$

39.) $\sqrt{-x} = 4x^3 - 5x$

40.) $x^2(x-7) = x(x-7)^2$

For each equation given in #41-45, decide whether dividing by the function $x - 7$ on both sides of the equation would result in an equivalent equation. For each of these, you'll want to check whether $x - 7$ has zeros in the implied domain of the given equation.

41.) $3x^2 - 5 = 2x + 8$

42.) $\log_e(x) = 3x - 5$

43.) $\frac{1}{x-7} = 3x$

44.) $\sqrt{-x} = 4x^3 - 5x$

45.) $x^2(x-7) = x(x-7)^2$

For #46-48, match the numbered equations on the left to the lettered equations on the right that they are equivalent to by multiplication.

46.) $x^2 + x = 3$	A.) $\frac{5-\sqrt{x}}{x+2} = \frac{3x+2}{x+2}$
47.) $\log_e(x) = 2$	B.) $\frac{1}{x}\log_e(x) = \frac{2}{x}$
48.) $5 - \sqrt{x} = 3x + 2$	C.) $e^x(x^2 + x) = 3e^x$

For #49-52, match the numbered equations on the left to the lettered equations on the right that they are equivalent to by invertible function.

49.) $e^{3x+5} = e^{2x-3}$	A.) $3x + 5 = x^{\frac{2}{3}}$
50.) $\log_e(x^2 + 1) = 2$	B.) $x^2 - 3x + 1 = 8x^3$
51.) $(3x + 5)^3 = x^2$	C.) $3x + 5 = 2x - 3$
52.) $\sqrt[3]{x^2 - 3x + 1} = 2x$	D.) $x^2 + 1 = e^2$

Find the set of solutions of each of the equations given in #53-61.

53.) $x^2 - 3 = x^2 + x$

54.) $\log_e(x^2) = 2$ where $x \in [0, \infty)$

55.) $4x^2 + 2x + 3 = 3x^2 + 2x + 2$

56.) $\sqrt[3]{4x + 2} = 4$

57.) $3x^2 - 6 = x^2 - 4x$ with $x > 10$

58.) $e^{3x+3} = 1$

59.) $x^5(x^2 + 3x - 2) = 0$ where $x < 0$

60.) $(\log_2(2x + 1))^3 = 8$

61.) $\frac{455}{x^2-9} = 5$ where $x < 5$