Three pairs of equivalent equations

① Equivalent by addition

- $f(x) + h(x) = g(x)$ with domain D
- $f(x) = g(x) - h(x)$ with domain D

② Equivalent by multiplication

- $h(x)\,f(x) = g(x)$ with domain D
- $f(x) = \dfrac{g(x)}{h(x)}$ with domain D

(If $h(x)$ has no zeros in D)

③ Equivalent by invertible function

- $h(f(x)) = g(x)$ with domain D
- $f(x) = h^{-1}(g(x))$ with domain D

Fact: Equivalent equations have the same domains and sets of solutions.

To find solutions of equations:

Ⓘ Write down the domain of your equation.

Ⓘⓘ Construct a sequence of equivalent equations that terminates in an easy equation to find solutions for.

Ⓘⓘⓘ The solutions of the final equation, with the domain from Ⓘ, are the solutions of the original equation.

Note: If you ever have an equation of polynomials, subtract a polynomial to obtain $p(x)=0$, then find the roots of $p(x)$.

Example: The equation $2x^2-x=5x+7$ is equivalent by addition to the equation $2x^2-x-(5x+7)=5x+7-(5x+7)$, which simplifies to $2x^2-6x-7=0$. We can find the solutions to $2x^2-6x-7=0$ by finding the roots of $2x^2-6x-7$. That will give us the solutions of $2x^2-x=5x+7$.