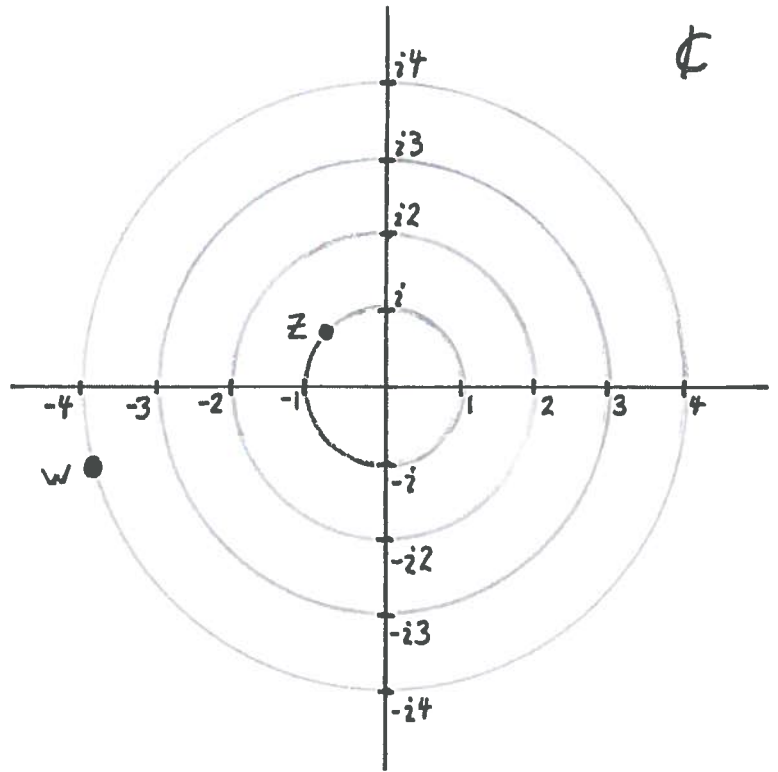# Polar Coordinates and Multiplication
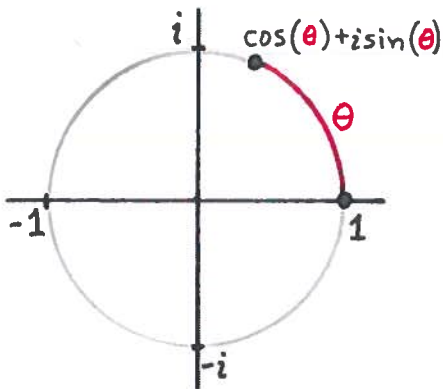
I. Multiplying a complex number by a real number scales the complex number.

1.) Put a dot on $3z$.

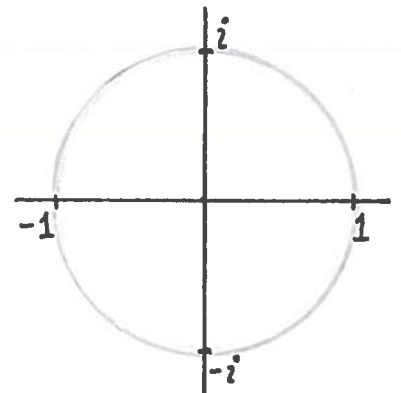2.) Put an X on $\frac{1}{2}w$.

II. The unit circle in the complex plane are the numbers $\cos(\theta) + i\sin(\theta)$ where $\theta \in \mathbb{R}$.
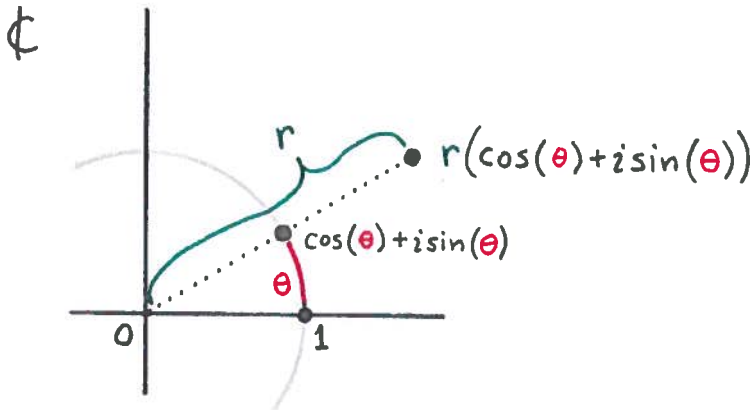

3.) Put a dot on
$$\cos\left(\frac{\pi}{3}\right) + i\sin\left(\frac{\pi}{3}\right)$$

4.) Put an X on
$$\cos\left(-\frac{\pi}{2}\right) + i\sin\left(-\frac{\pi}{2}\right)$$

<u>III.</u> Any complex number can be written as

$$r(\cos(\theta) + i\sin(\theta)) \quad \text{where} \quad r \geq 0 \quad \text{and} \quad \theta \in \mathbb{R}.$$

- r is the norm of the complex number, its distance from 0.

- θ is the angle between the complex number and 1, measured at 0.

5.) Write $2 + i3$ in polar coordinates.

6.) Write $3 + i4$ in polar coordinates.

7.) Put a dot on

$2\left(\cos\left(\frac{2\pi}{3}\right) + i\sin\left(\frac{2\pi}{3}\right)\right)$.
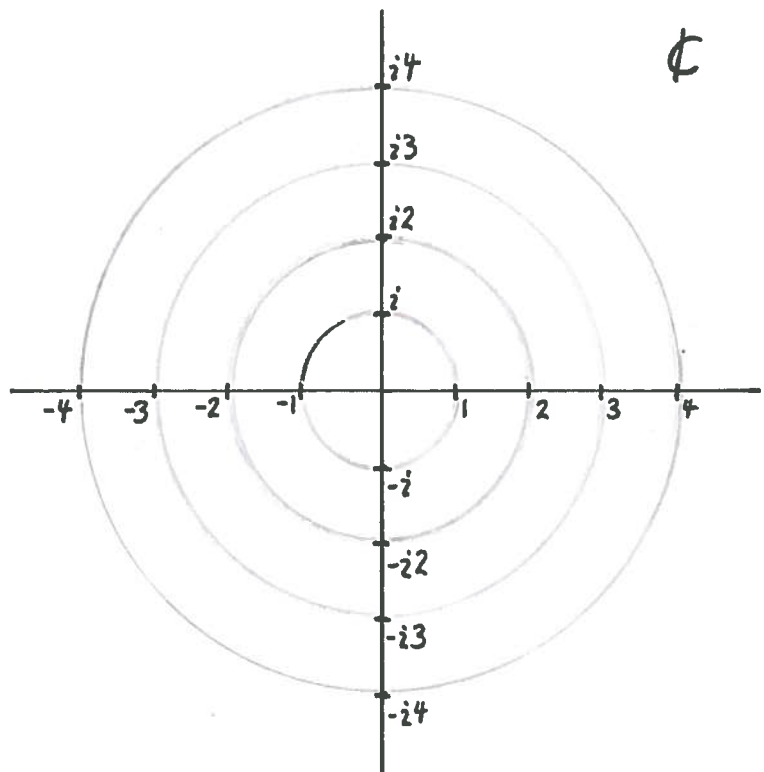
8.) Put an X on

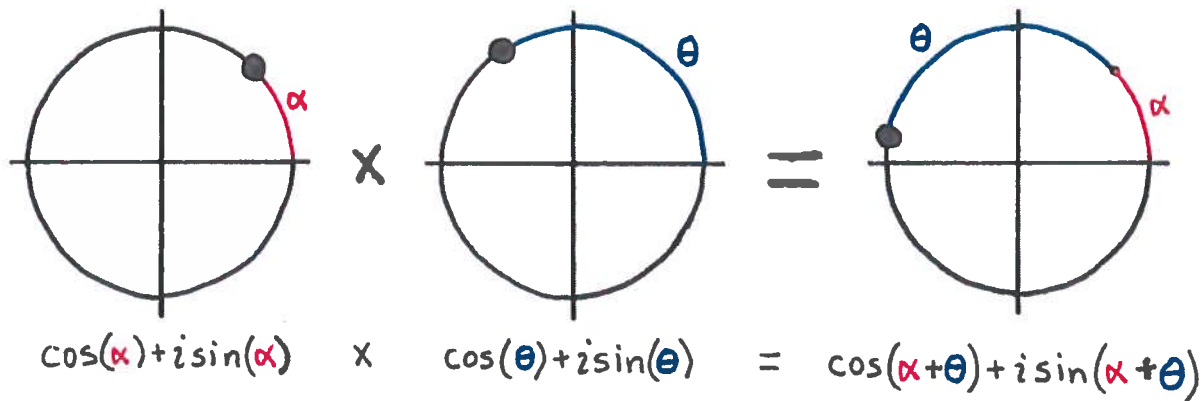$4\left(\cos\left(\frac{3\pi}{2}\right) + i\sin\left(\frac{3\pi}{2}\right)\right)$.

9.) Put a △ on

$\cos\left(-\frac{\pi}{4}\right) + i\sin\left(-\frac{\pi}{4}\right)$.
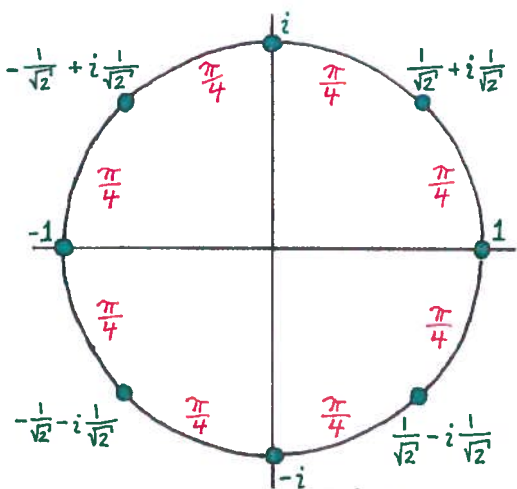
10.) Put a ☐ on

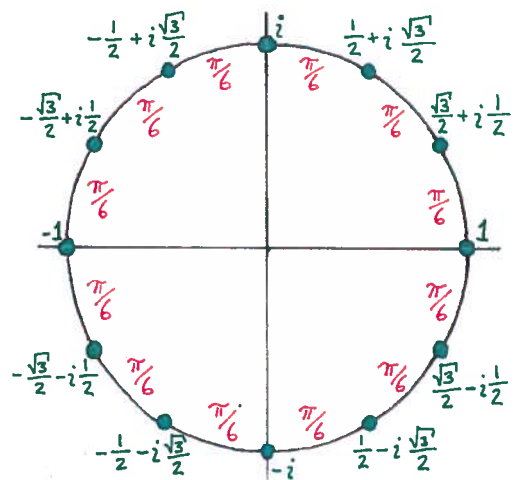$3\left(\cos\left(\frac{5\pi}{4}\right) + i\sin\left(\frac{5\pi}{4}\right)\right)$.

<u>IV.</u> To multiply complex numbers on the unit circle, add their angles.

$$\cos(\alpha)+i\sin(\alpha) \quad \times \quad \cos(\theta)+i\sin(\theta) \quad = \quad \cos(\alpha+\theta)+i\sin(\alpha+\theta)$$

11.) Find $\left[\cos(4)+i\sin(4)\right]\left[\cos(3)+i\sin(3)\right]$

12.) Find $\left[\cos(2)+i\sin(2)\right]\left[\cos(13)+i\sin(13)\right]$

13.) Find $\left(\frac{\sqrt{3}}{2}+i\frac{1}{2}\right)\left(-\frac{\sqrt{3}}{2}-i\frac{1}{2}\right)$

14.) Find $\left(-\frac{1}{\sqrt{2}}+i\frac{1}{\sqrt{2}}\right)\left(-\frac{1}{\sqrt{2}}-i\frac{1}{\sqrt{2}}\right)$

15.) Find $\left(\frac{\sqrt{3}}{2}+i\frac{1}{2}\right)^{7}$

16.) Find $\left(\frac{1}{\sqrt{2}}+i\frac{1}{\sqrt{2}}\right)^{6}$

The 8   8th roots of unity

The 12   12th roots of unity

<u>V</u>. To multiply complex numbers, multiply their
norms and add their angles.

17.) Find $2\left(\cos(3)+i\sin(3)\right)\,4\left(\cos(7)+i\sin(7)\right)$.

18.) Find $3\left(\cos(8)+i\sin(8)\right)\,7\left(\cos(5)+i\sin(5)\right)$.

<u>VI</u>. Multiplying by $r\left(\cos(\theta)+i\sin(\theta)\right)$ scales
by r and rotates by θ.

19.) Put a dot on

$3\left(\cos(\pi)+i\sin(\pi)\right)z$.

20.) Put an X on

$\left(\cos\left(-\frac{\pi}{2}\right)+i\sin\left(-\frac{\pi}{2}\right)\right)z$.

21.) Put a Δ on

$\left(\cos\left(\frac{\pi}{6}\right)+i\sin\left(\frac{\pi}{6}\right)\right)w$.

22.) Put a $\square$ on

$\frac{1}{2}\left(\cos\left(\frac{\pi}{4}\right)+i\sin\left(\frac{\pi}{4}\right)\right)w$.