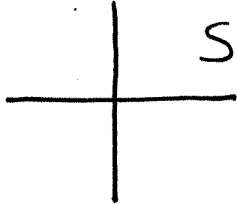


Classification of Conics

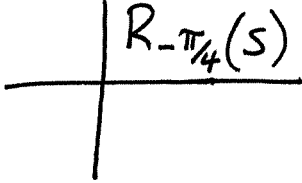
A. Rotate to remove xy -term


$$4x^2 + 2xy + 4y^2 + \sqrt{2}x + 5\sqrt{2}y + 4 = 0$$

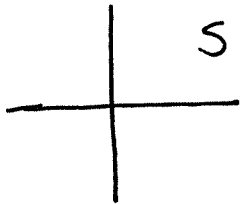
$R_{-\pi/4}$

$$R_{\pi/4} = \begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

$x \mapsto \frac{x-y}{\sqrt{2}}$
 $y \mapsto \frac{x+y}{\sqrt{2}}$



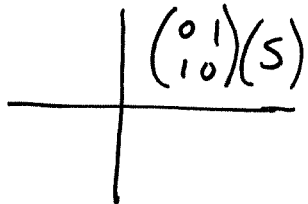
B. Flip over to have an x^2 -term


$$2y^2 + 3x + 4y + 1 = 0$$

$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

$x \mapsto y$
 $y \mapsto x$



C. Shift horizontally to remove x-term

$$\begin{array}{ccc} \begin{array}{|c} S \\ \hline \end{array} & \xrightarrow{A_{(5,0)}} & \begin{array}{|c} A_{(5,0)}(S) \\ \hline \end{array} \\ x^2 + 3y^2 + 10x + 4y + 2 = 0 & \xrightarrow[A_{(-5,0)}]{\begin{array}{l} x \mapsto x-5 \\ y \mapsto y \end{array}} & \end{array}$$

D. If there's a y^2 -term, shift vertically to remove y-term

$$\begin{array}{ccc} \begin{array}{|c} S \\ \hline \end{array} & \xrightarrow{A_{(0,1)}} & \begin{array}{|c} A_{(0,1)}(S) \\ \hline \end{array} \\ 3x^2 + 2y^2 + 4y + 5 = 0 & \xrightarrow[A_{(0,-1)}]{\begin{array}{l} x \mapsto x \\ y \mapsto y-1 \end{array}} & \end{array}$$

E. Equivalent equation has constant term 0 or 1

Multiply both sides of $4x^2 + 8y^2 + 2 = 0$ by $\frac{1}{2}$.

F. Scale x-axis so that the x^2 coefficient is 1 or -1

$$\begin{array}{ccc} \begin{array}{c} | \quad S \\ \hline \\ | \\ 25x^2 + 7y^2 + 1 = 0 \end{array} & \begin{array}{c} \xrightarrow{\begin{pmatrix} 5 & 0 \\ 0 & 1 \end{pmatrix}} \\ \xrightarrow{\begin{pmatrix} \frac{1}{5} & 0 \\ 0 & 1 \end{pmatrix}} \\ x \mapsto \frac{x}{5} \\ y \mapsto y \end{array} & \begin{array}{c} | \quad \begin{pmatrix} 5 & 0 \\ 0 & 1 \end{pmatrix} (S) \\ \hline \\ | \end{array} \end{array}$$

G. Scale y-axis so that the y^2 coefficient is 0, 1, or -1

$$\begin{array}{ccc} \begin{array}{c} | \quad S \\ \hline \\ | \\ x^2 - 4y^2 + 1 = 0 \end{array} & \begin{array}{c} \xrightarrow{\begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix}} \\ \xrightarrow{\begin{pmatrix} 1 & 0 \\ 0 & \frac{1}{2} \end{pmatrix}} \\ x \mapsto x \\ y \mapsto \frac{y}{2} \end{array} & \begin{array}{c} | \quad \begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix} (S) \\ \hline \\ | \end{array} \end{array}$$

H. Scale y-axis so y-coefficient is 0 or 1

$x^2 - 4y + 1 = 0$

$$\begin{pmatrix} 1 & 0 \\ 0 & -4 \end{pmatrix}$$
$$\begin{pmatrix} 1 & 0 \\ 0 & \frac{1}{-4} \end{pmatrix}$$

$x \mapsto x$
 $y \mapsto -\frac{y}{4}$

$(1 \ 0) (0 \ -4) (S)$

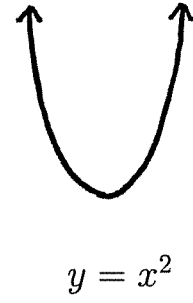
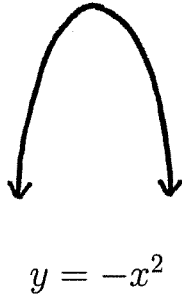
I. If there's a y-term, shift vertically to remove constant

$x^2 + y + 1 = 0$

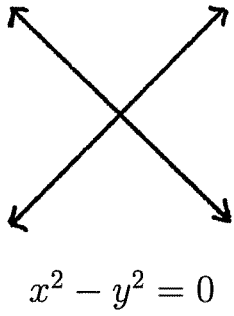
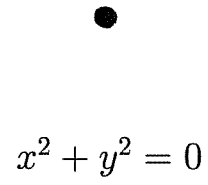
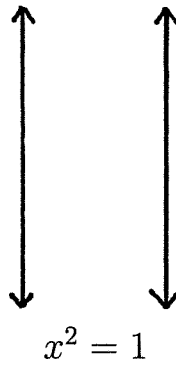
$$A_{(0,1)}$$
$$A_{(0,-1)}$$

$x \mapsto x$
 $y \mapsto y - 1$

$A_{(0,1)}(S)$



$x^2 = -1$



$x^2 + y^2 = -1$

