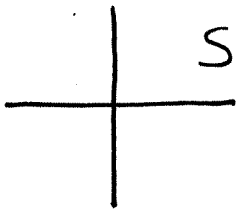
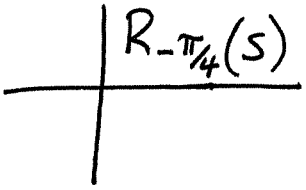
Classification of Conics

A. Rotate to remove xy-term



$R_{-\pi/4}$



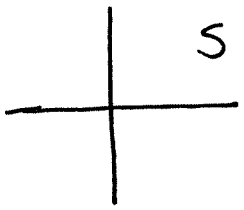
$4x^2+2xy+4y^2+\sqrt{2}x+5\sqrt{2}y+4=0$

$R_{\pi/4}=\begin{pmatrix}\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}}\\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}}\end{pmatrix}$

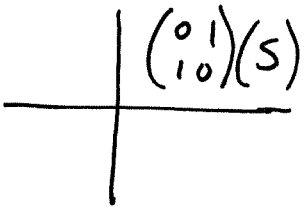
$x\mapsto \frac{x-y}{\sqrt{2}}$

$y\mapsto \frac{x+y}{\sqrt{2}}$

B. Flip over to have an x^2-term



$\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}$



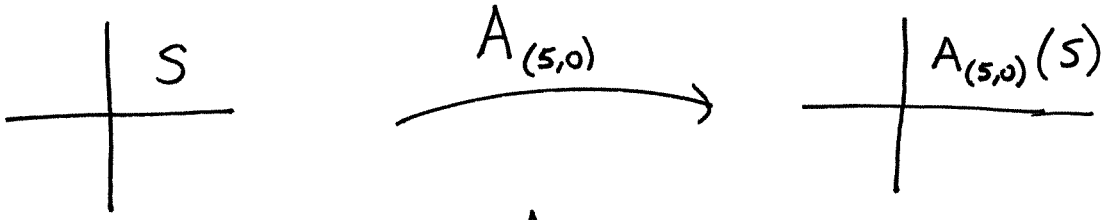
$2y^2+3x+4y+1=0$

$\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}$

$x\mapsto y$

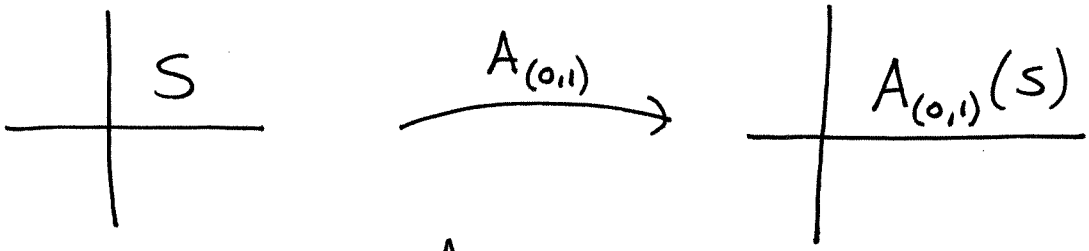
$y\mapsto x$

C. Shift horizontally to remove x-term



$x^2+3y^2+10x+4y+2=0$ $\xrightarrow[x \mapsto x-5,\ y \mapsto y]{A_{(-5,0)}}$

D. If there's a y^2-term, shift vertically to remove y-term

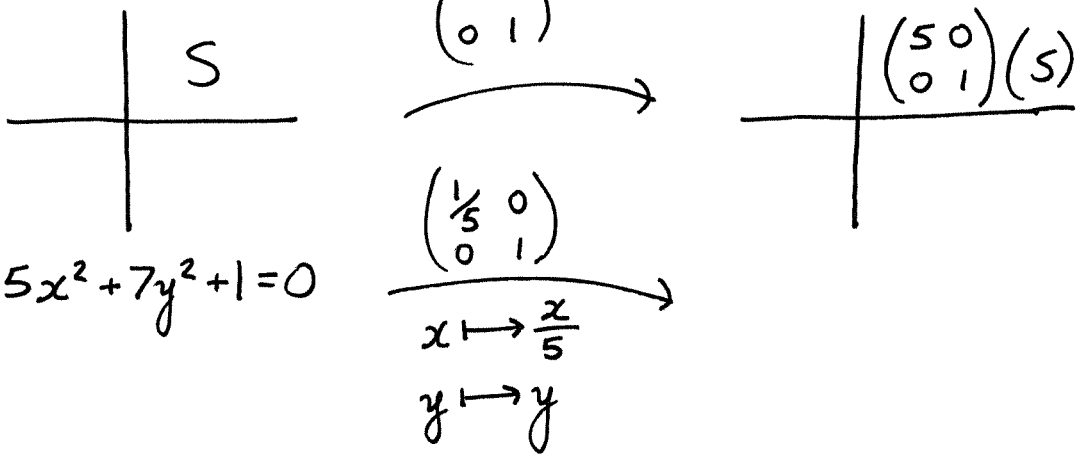


$3x^2+2y^2+4y+5=0$ $\xrightarrow[x \mapsto x,\ y \mapsto y-1]{A_{(0,-1)}}$

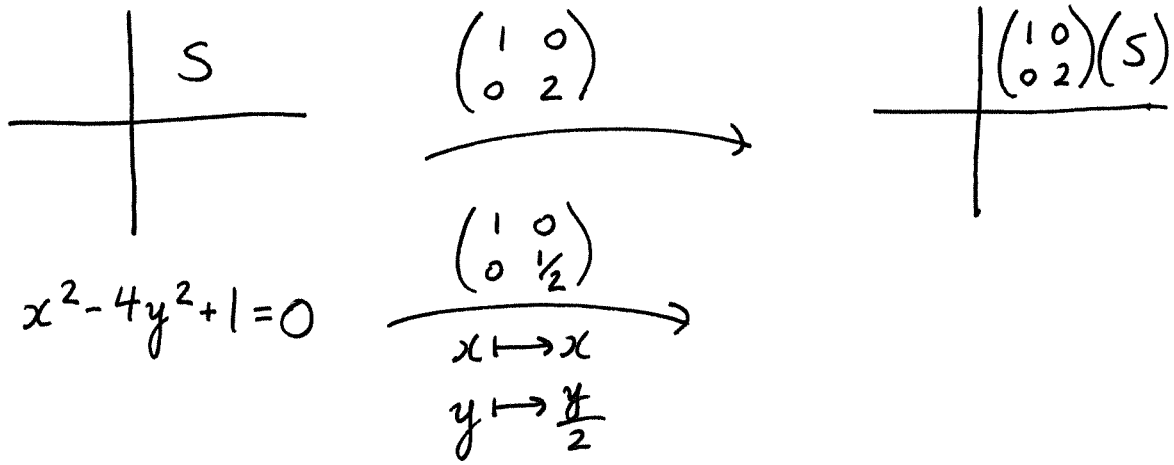
E. Equivalent equation has constant term 0 or 1

Multiply both sides of $4x^2+8y^2+2=0$ by $\frac{1}{2}$.

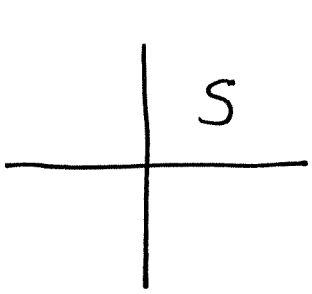
F. Scale x-axis so that the x^2 coefficient is 1 or -1



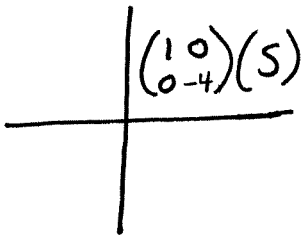
G. Scale y-axis so that the y^2 coefficient is 0, 1, or -1



H. Scale y-axis so y-coefficient is 0 or 1



$\begin{pmatrix} 1 & 0 \\ 0 & -4 \end{pmatrix}$



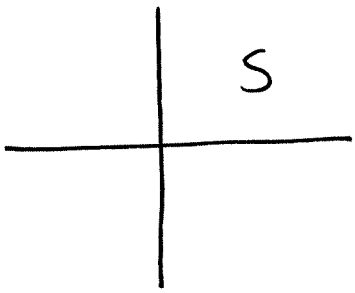
$x^2 - 4y + 1 = 0$

$\begin{pmatrix} 1 & 0 \\ 0 & \frac{1}{-4} \end{pmatrix}$

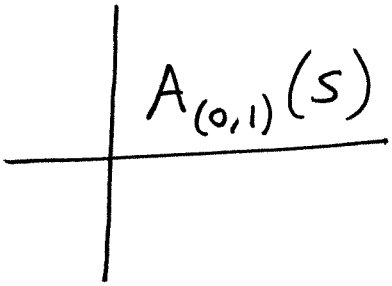
$x \longmapsto x$

$y \longmapsto -\frac{y}{4}$

I. If there's a y-term, shift vertically to remove constant



$A_{(0,1)}$



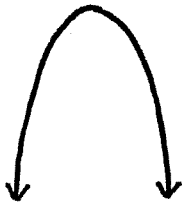
$x^2 + y + 1 = 0$

$A_{(0,-1)}$

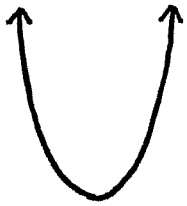
$x \longmapsto x$

$y \longmapsto y - 1$

$x^2 = 0$

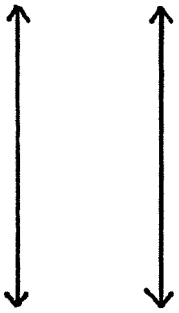


$y = -x^2$



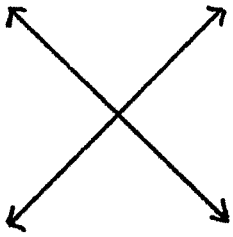
$y = x^2$

$x^2 = -1$



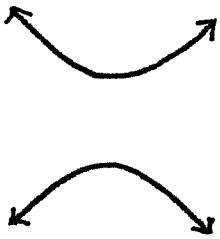
$x^2 = 1$

$x^2 + y^2 = 0$

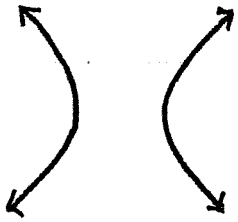


$x^2 - y^2 = 0$

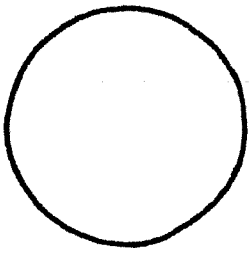
$x^2 + y^2 = -1$



$y^2 - x^2 = 1$



$x^2 - y^2 = 1$



$x^2 + y^2 = 1$