Planar Transformations of Graphs

$(d>0\ ,\ c>1)$

Start function : $f(x)$

Change "outside the function"

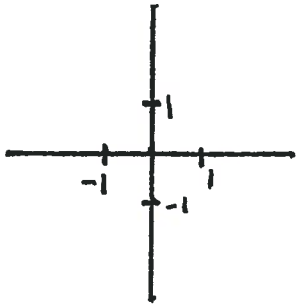
New function (algebra)	How the graph changes (geometry)
$f(x)+d$	$y \mapsto y+d$, up d
$f(x)-d$	$y \mapsto y-d$, down d
$cf(x)$	$y \mapsto cy$, vertical stretch by c
$\frac{1}{c} f(x)$	$y \mapsto \frac{y}{c}$, vertical shrink by $\frac{1}{c}$
$-f(x)$	$y \mapsto -y$, flip over x-axis

Change "inside the function"

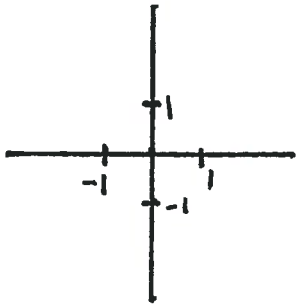
New function (algebra)	How the graph changes (geometry)
$f(x+d)$	$x \mapsto x-d$, left d
$f(x-d)$	$x \mapsto x+d$, right d
$f(cx)$	$x \mapsto \frac{x}{c}$, horizontal shrink by $\frac{1}{c}$
$f\left(\frac{x}{c}\right)$	$x \mapsto cx$, horizontal stretch by c
$f(-x)$	$x \mapsto -x$, flip over y-axis

Inverses

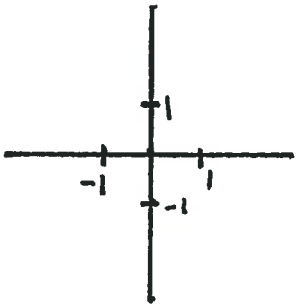
New function (algebra)	How the graph changes (geometry)
$f^{-1}(x)$	$\begin{pmatrix} x \mapsto y \\ y \mapsto x \end{pmatrix}$, flip over "$x=y$ line"



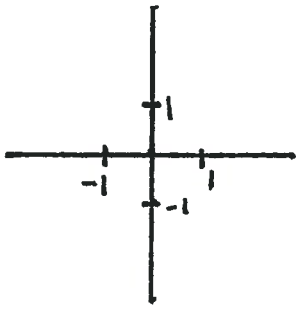
x^2



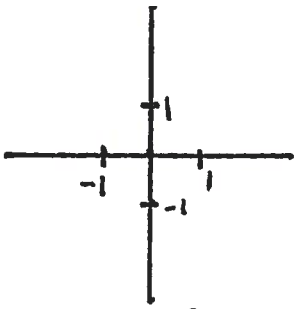
x^2+1



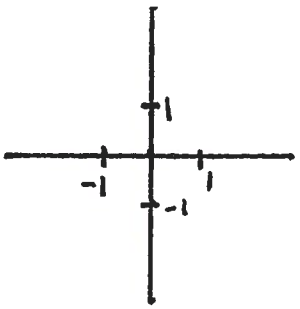
x^2-1



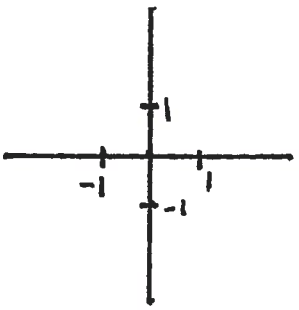
$(x+1)^2$



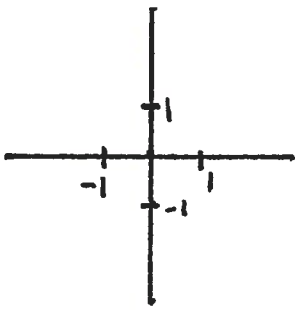
$(x-1)^2$



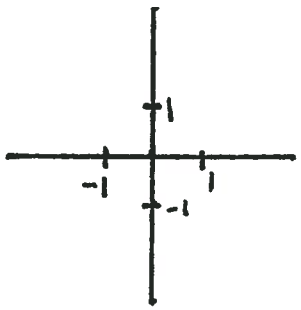
$(-x)^2$



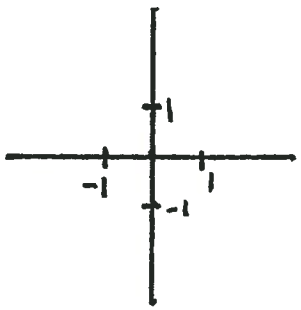
$-x^2$



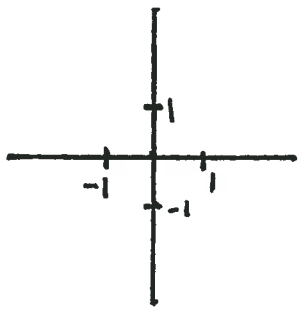
$-(x+1)^2$



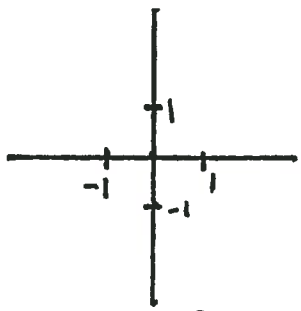
x^3



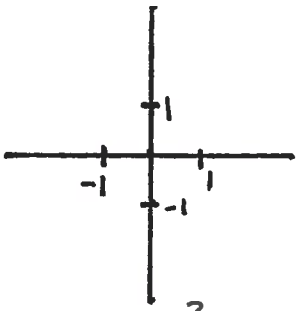
x^3+1



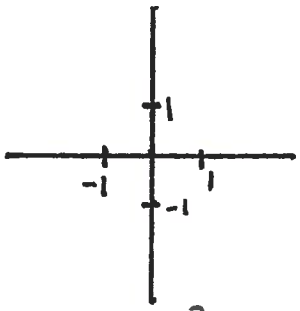
x^3-1



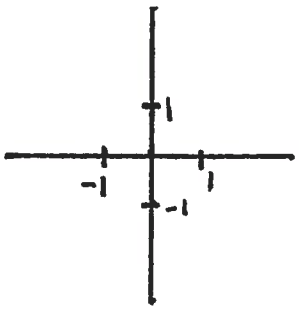
$(x+1)^3$



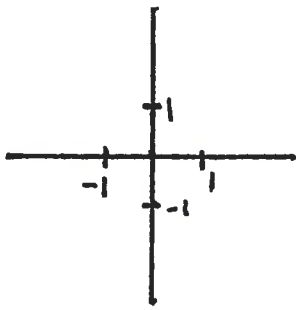
$(x-1)^3$



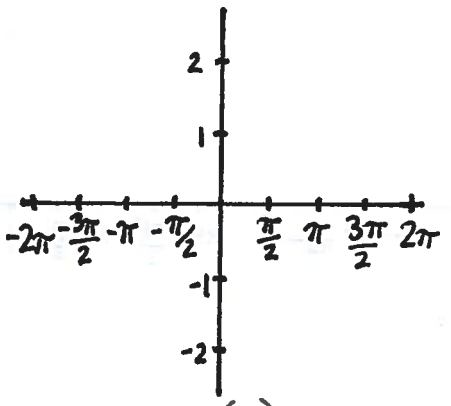
$(-x)^3$



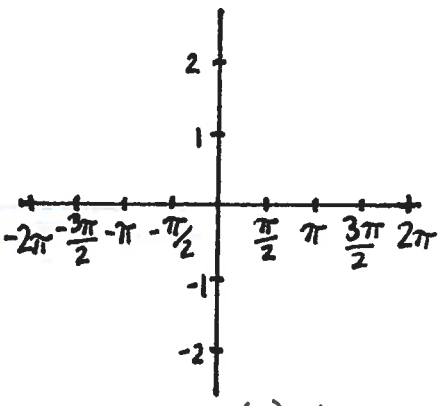
$-x^3$



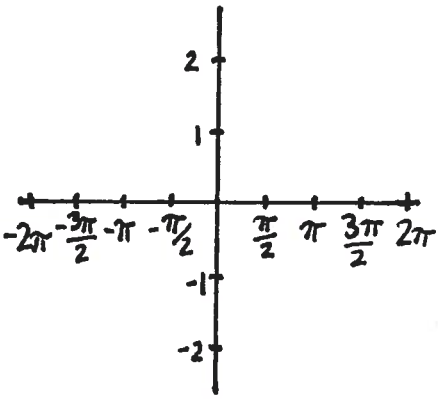
$\sqrt[3]{x}$



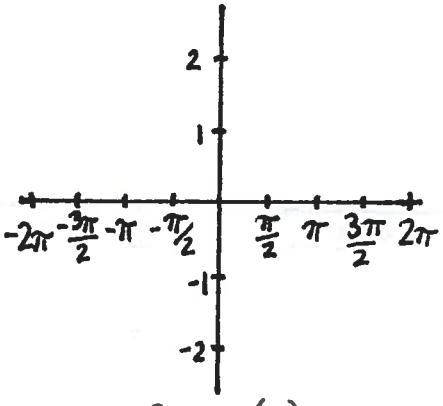
$\cos(x)$



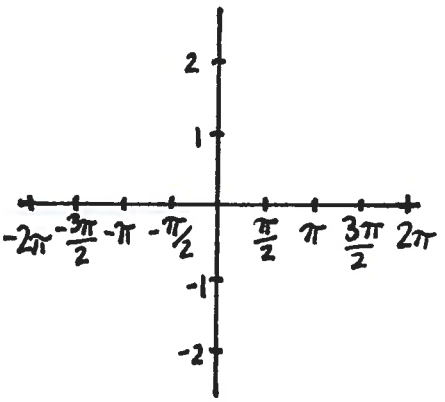
$\cos(x)+1$



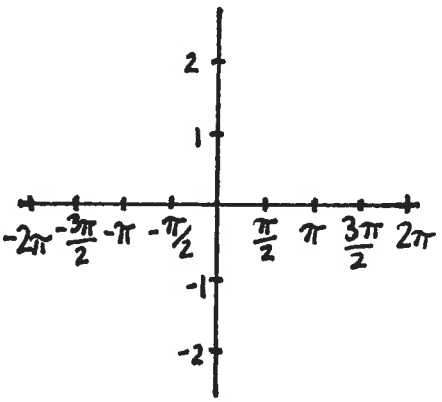
$\cos(x)-1$



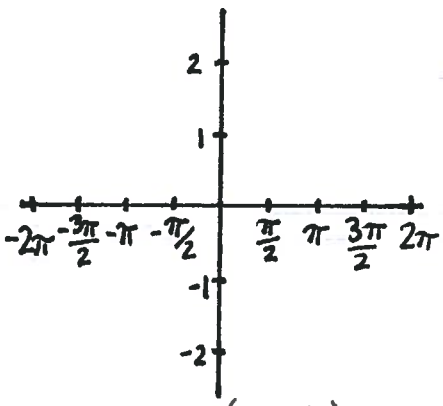
$2\cos(x)$



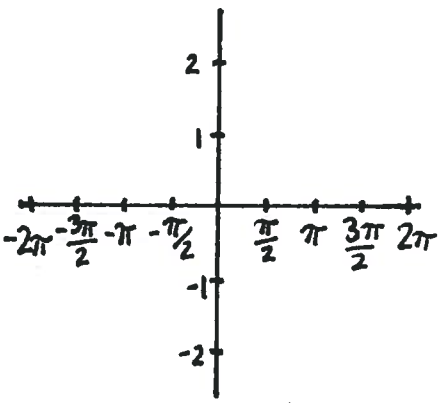
$\frac{1}{2}\cos(x)$



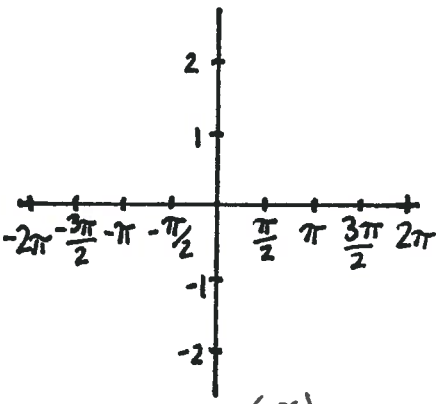
$\cos\left(x+\frac{\pi}{2}\right)$



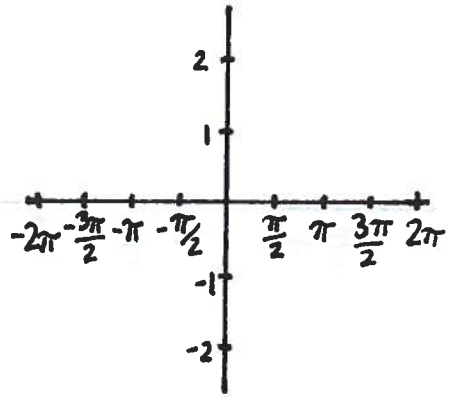
$\cos\left(x-\frac{\pi}{2}\right)$



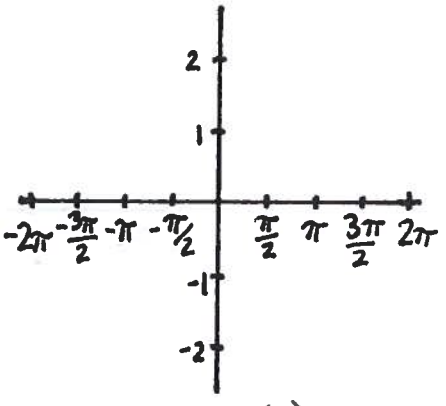
$\cos(2x)$



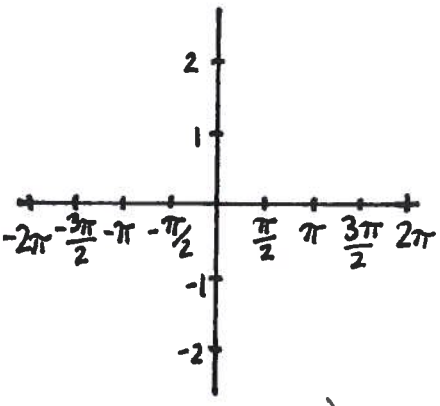
$\cos\left(\frac{x}{2}\right)$



$\cos(-x)$



$-\cos(x)$



$\cos(x+\pi)$