Practice First Midterm Exam

Conics

For #1-8, match the numbered quadratic equations in two variables with their lettered sets of solutions.

1.) $x^2 = -1$ G

2.) $x^2 = 0$ A

3.) $x^2 = 1$ D

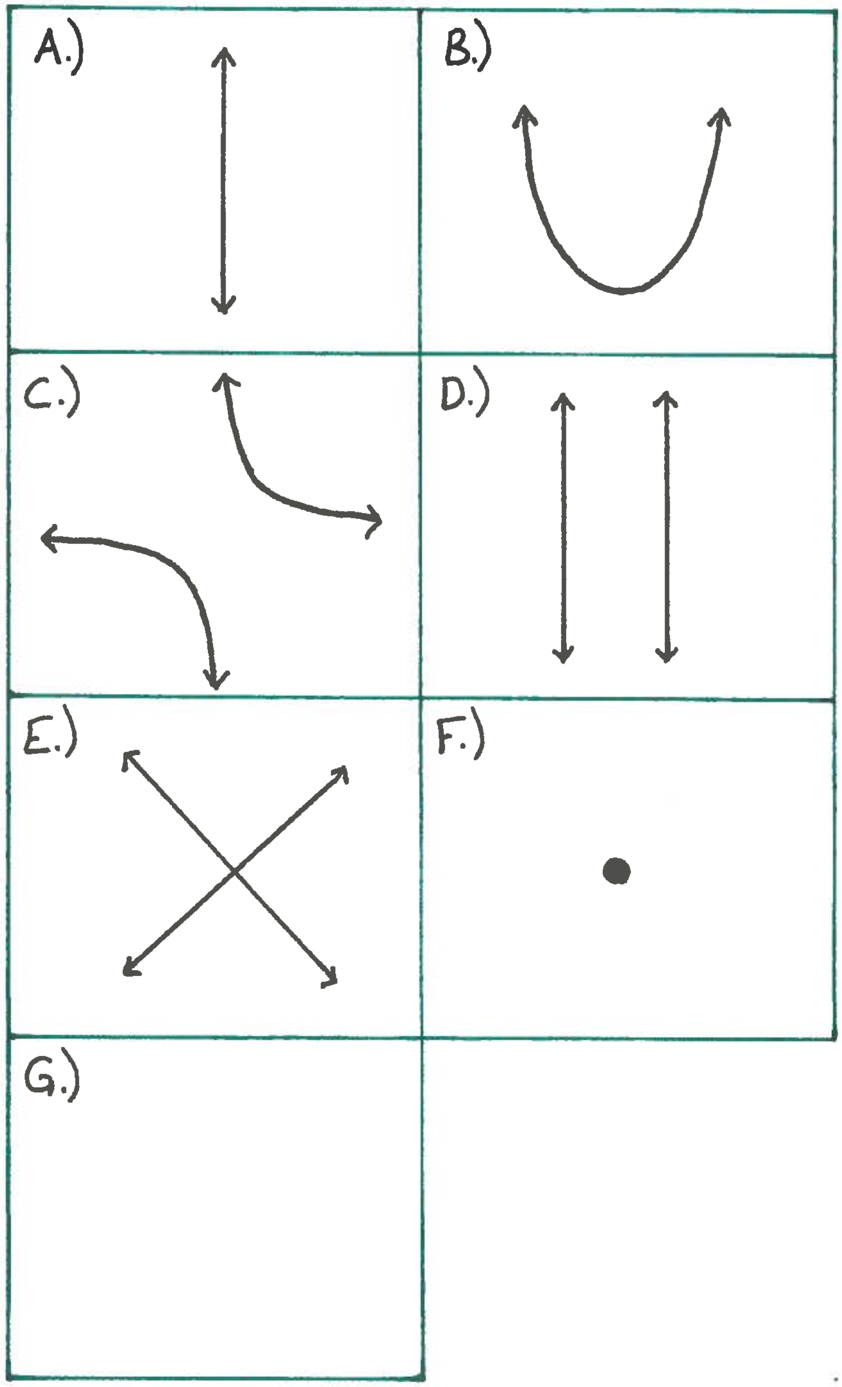
4.) $xy = 1$ C

5.) $y = x^2$ B

6.) $x^2 + y^2 = -1$ G

7.) $x^2 + y^2 = 0$ F

8.) $x^2 - y^2 = 0$ E



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Linear algebra

For #9-15, give the vector, written as a **ROW** vector.

9.) $A_{(-2,4)}(3,-5) = (3-2, -5+4) = (1,-1)$

10.) $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} -1 \\ 8 \end{pmatrix} = \begin{pmatrix} -1 \\ 8 \end{pmatrix} = (-1,8)$

$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ is the identity matrix. It doesn't change vectors.

11.) $\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 3 \\ 4 \end{pmatrix} = \begin{pmatrix} -3 \\ 4 \end{pmatrix} = (-3,4)$

$\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$ scales the x-coordinate by -1.

12.) $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} -5 \\ -2 \end{pmatrix} = \begin{pmatrix} -5 \\ 2 \end{pmatrix} = (-5,2)$

$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ scales the y-coordinate by -1.

13.) $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 3 \\ 4 \end{pmatrix} = \begin{pmatrix} 4 \\ 3 \end{pmatrix} = (4,3)$

$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ switches the x- and y-coordinates.

14.) $\begin{pmatrix} 3 & 0 \\ 0 & 2 \end{pmatrix} \begin{pmatrix} -1 \\ 6 \end{pmatrix} = \begin{pmatrix} 3(-1) \\ 2(6) \end{pmatrix} = \begin{pmatrix} -3 \\ 12 \end{pmatrix} = (-3,12)$

$\begin{pmatrix} 3 & 0 \\ 0 & 2 \end{pmatrix}$ scales the x-coordinate by 3 and scales the y-coordinate by 2.

15.) $\begin{pmatrix} 2 & -4 \\ -3 & 6 \end{pmatrix} \begin{pmatrix} 3 \\ -2 \end{pmatrix} = \begin{pmatrix} (2)(3)+(-4)(-2) \\ (-3)(3)+(6)(-2) \end{pmatrix} = \begin{pmatrix} 6+8 \\ -9-12 \end{pmatrix} = \begin{pmatrix} 14 \\ -21 \end{pmatrix} = (14,-21)$

16.) Find the product $\begin{pmatrix} -2 & 4 \\ 1 & 3 \end{pmatrix} \begin{pmatrix} -5 & 1 \\ -2 & 3 \end{pmatrix} = \begin{pmatrix} (-2)(-5)+(4)(-2) & (-2)(1)+(4)(3) \\ (1)(-5)+(3)(-2) & (1)(1)+(3)(3) \end{pmatrix}$

$$= \begin{pmatrix} 10-8 & -2+12 \\ -5-6 & 1+9 \end{pmatrix}$$

$$= \begin{pmatrix} 2 & 10 \\ -11 & 10 \end{pmatrix}$$

17.) Give the determinant of $\begin{pmatrix} 2 & 8 \\ 7 & 3 \end{pmatrix}$

$(2)(3)-(8)(7) = 6-56 = -50$

18.) Give the inverse of $\begin{pmatrix} 3 & 8 \\ -9 & -20 \end{pmatrix}$

determinant is $(3)(-20)-(8)(-9) = -60+72 = 12.$

Thus,

$$\begin{pmatrix} 3 & 8 \\ -9 & -20 \end{pmatrix}^{-1} = \frac{1}{12}\begin{pmatrix} -20 & -8 \\ 9 & 3 \end{pmatrix} = \begin{pmatrix} \frac{-20}{12} & \frac{-8}{12} \\ \frac{9}{12} & \frac{3}{12} \end{pmatrix} = \begin{pmatrix} -\frac{5}{3} & -\frac{2}{3} \\ \frac{3}{4} & \frac{1}{4} \end{pmatrix}$$

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Lines

19.) Give an equation for a line in the plane that has slope -6 and passes through the point $(0, 0)$.

$y = -6x$

20.) Give an equation for a line in the plane that has slope 3 and passes through the point $(4, 8)$.

$(y-8) = 3(x-4)$

21.) Give the slope of the line that passes through the points $(2,-4)$ and $(-3,2)$.

$$\frac{2-(-4)}{-3-2} = \frac{2+4}{-5} = -\frac{6}{5}$$

22.) Give an equation for the line that passes through the points $(2,-4)$ and $(-3,2)$. There are two different good answers here. You'd just want to give one of these two answers.

① Using the point $(2,-4)$: $(y-(-4)) = -\frac{6}{5}(x-2)$

which is $(y+4) = -\frac{6}{5}(x-2)$

② Using the point $(-3,2)$: $(y-2) = -\frac{6}{5}(x-(-3))$

which is $(y-2) = -\frac{6}{5}(x+3)$

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Equations in One Variable

23.) Give the implied domain of the equation $48x^2+3x+\sqrt{x}=e^x+7$.

We can only take the square root of a number if it's not negative. Thus, the implied domain is $[0,\infty)$.

For #24-26, find the solutions of the given equations, and explain your answers. #24-26 are worth 2 points each.

24.) $(e^x)^2 + 2e^x - 3 = 0$

⓪ The implied domain is $\mathbb{R}$ because any number can be placed in an exponent of e.

① This is a quadratic equation in e^x, an equation of the form $a(e^x)^2+b(e^x)+c=0$. Here, $a=1$, $b=2$, and $c=-3$. The discriminant is $b^2-4ac=4+12=16$. Since 16 is positive, there are two solutions for e^x. Either

$e^x = \frac{-2-\sqrt{16}}{2} = \frac{-2-4}{2} = \frac{-6}{2} = -3$ or

$e^x = \frac{-2+\sqrt{16}}{2} = \frac{-2+4}{2} = \frac{2}{2} = 1$.

② If $e^x=-3$, then there are no solutions because e^x can only be positive.

If $e^x=1$, then $x=\log_e(1)=0$.

Therefore, $\boxed{x=0}$ is the only solution of $(e^x)^2+2(e^x)-3=0$

③

$$(e^x)^2+2(e^x)-3=0 \quad \begin{cases} \to e^x=-3 \longrightarrow \text{no solution} \\ \to e^x=1 \longrightarrow x=0 \end{cases}$$

25.) $x\log_e(3x-2) = x$ where $x > \frac{2}{3}$

Since $x>2/3$, we know $x\neq 0$. Thus, we can divide by x:

$\log_e(3x-2) = \frac{x}{x} = 1$

$\Rightarrow$

$3x-2=e^1=e$

$\Rightarrow$

$3x=e+2$

$\Rightarrow x=\frac{e+2}{3}$

Notice that $\frac{e+2}{3} > \frac{2}{3}$, since $e>0$, so $\boxed{x=\frac{e+2}{3}}$ is a solution to this equation. It's the only solution.

26.) $\sqrt{3x+2} = -1$

A square root cannot be negative.

Therefore, there is $\boxed{\text{no solution}}$.

On the actual exam you wouldn't have to write quite so much for #24 and #25, but you would need some explanation. The answers are boxed in red.

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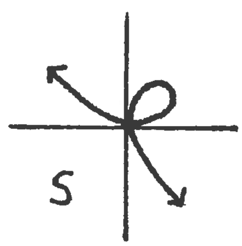
Equations in two variables and their solutions

27.) Suppose $p(x, y) = 2xy - 5y^2 - x + 11$. Find $p \circ A_{(2,-3)}(x, y)$. (You don't have to simplify your answer.)

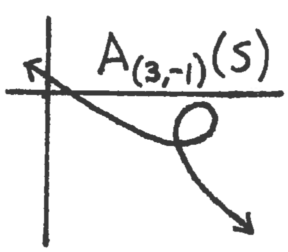
$$p \circ A_{(2,-3)}(x,y) = p(x+2, y-3)$$
$$= 2(x+2)(y-3) - 5(y-3)^2 - (x+2) + 11$$

#28-30 are worth 2 points each.

The "Folium of Descartes" is the set of solutions, S, of the polynomial equation $x^3 + y^3 = xy$.



28.) Give an equation for $A_{(3,-1)}(S)$, the Folium of Descartes shifted right 3 and down 1. (You don't have to simplify your answer.)

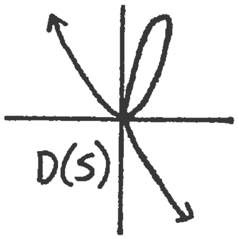


$$S \xrightarrow{A_{(3,-1)}} A_{(3,-1)}(S)$$

$$x^3 + y^3 = xy \xrightarrow{A_{(3,-1)}^{-1} = A_{(-3,1)}} (x-3)^3 + (y+1)^3 = (x-3)(y+1)$$

$$x \longmapsto x - 3$$
$$y \longmapsto y + 1$$

29.) Let $D = \begin{pmatrix} \frac{1}{2} & 0 \\ 0 & 4 \end{pmatrix}$. Give an equation for $D(S)$, the Folium of Descartes scaled by $\frac{1}{2}$ in the x-coordinate and 4 in the y-coordinate. (You don't have to simplify your answer.)

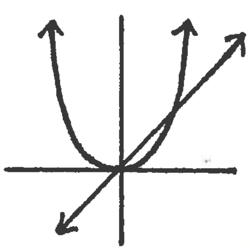


$$S \xrightarrow{\begin{pmatrix} 1/2 & 0 \\ 0 & 4 \end{pmatrix}} D(S)$$

$$\begin{pmatrix} 1/2 & 0 \\ 0 & 4 \end{pmatrix}^{-1} = \begin{pmatrix} 2 & 0 \\ 0 & 1/4 \end{pmatrix}$$

$$x^3 + y^3 = xy \xrightarrow[\substack{x \longmapsto 2x \\ y \longmapsto \frac{y}{4}}]{} (2x)^3 + \left(\frac{y}{4}\right)^3 = (2x)\left(\frac{y}{4}\right)$$

30.) Give an equation whose set of solutions is the union of the parabola $y = x^2$ and the line $x = y$. (You don't have to simplify your answer.)



$y = x^2$ is equivalent to $y - x^2 = 0$

$x = y$ is equivalent to $x - y = 0$

Therefore, $(y - x^2)(x - y) = 0$ is an equation for the union of the parabola and the line.

<u>Note:</u> $(x^2 - y)(x - y) = 0$, $(y - x^2)(y - x) = 0$, and $(x^2 - y)(y - x) = 0$ are also good answers for #30. You'd just write one of these on an actual exam.

First Name:________________ Last Name:________________

1.) G

2.) A

3.) D

4.) C

5.) B

6.) G

7.) F

8.) E

9.) $(1,-1)$

10.) $(-1,8)$

11.) $(-3,4)$

12.) $(-5,2)$

13.) $(4,3)$

14.) $(-3,12)$

15.) $(14,-21)$

16.) $\begin{pmatrix} 2 & 10 \\ -11 & 10 \end{pmatrix}$

17.) -50

18.) $\begin{pmatrix} -\frac{5}{3} & -\frac{2}{3} \\ \frac{3}{4} & \frac{1}{4} \end{pmatrix}$

19.) $y=-6x$

20.) $(y-8)=3(x-4)$

21.) $-\frac{6}{5}$

22.) $(y+4)=-\frac{6}{5}(x-2)$, or $(y-2)=-\frac{6}{5}(x+3)$

23.) $[0,\infty)$

There are two different good answers for #22. Both are written above. On the actual exam, you'd just write one of them