# Equations in Two Variables

Geometry and algebra have an inverse relationship

1.)

$$x^2 + y^2 = 1$$

right 2
up 1

$x \longmapsto x+2$
$y \longmapsto y+1$

$x \longmapsto x-2$
$y \longmapsto y-1$
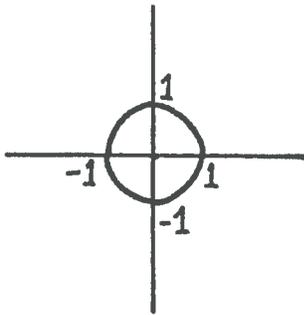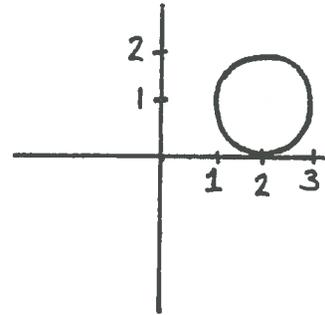
$$(x-2)^2 + (y-1)^2 = 1$$

2.)
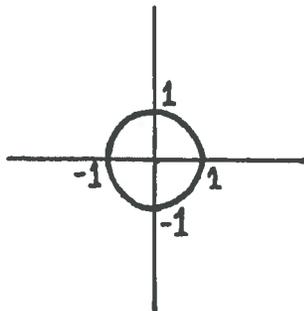
$$x^2 + y^2 = 1$$

scale x by 2
scale y by 3

$x \longmapsto 2x$
$y \longmapsto 3y$

$x \longmapsto \frac{x}{2}$
$y \longmapsto \frac{y}{3}$
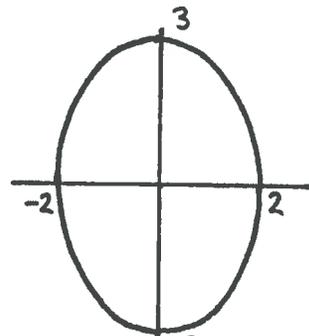
$$\left(\frac{x}{2}\right)^2 + \left(\frac{y}{3}\right)^2 = 1$$

or

$$\frac{x^2}{4} + \frac{y^2}{9} = 1$$

3.)
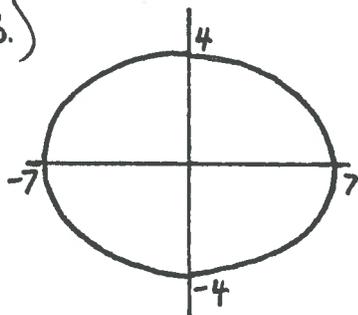
$$\frac{x^2}{49} + \frac{y^2}{16} = 1$$

right 2
up 1

$x \longmapsto x+2$
$y \longmapsto y+1$

$x \longmapsto x-2$
$y \longmapsto y-1$
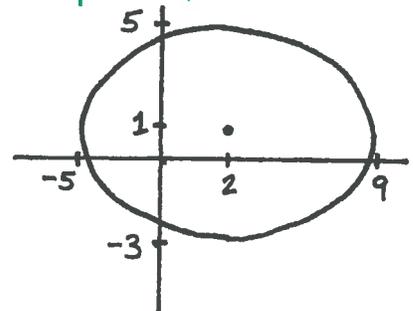
$$\frac{(x-2)^2}{49} + \frac{(y-1)^2}{16} = 1$$

Transformations of Graphs

(•) x goes inside a function, y comes out. $y \leftarrow f(\ \underset{\downarrow}{\overset{x}{\ }}\ ) \rightarrow y$

In the first blank next to each function, write an X if the change from $f(x)$ is in the x-coordinate. Write a Y if the change is in the y-coordinate.

4.) $f(x+1)$ __X__ , __H__   5.) $2f(x)$ __Y__ , __A__   6.) $f(\frac{x}{2})$ __X__ , __C__

7.) $-f(x)$ __Y__ , __I__   8.) $f(x)-1$ __Y__ , __E__   9.) $f(x-1)$ __X__ , __F__

10.) $f(x)+1$ __Y__ , __B__   11.) $f(2x)$ __X__ , __D__   12.) $\frac{1}{2}f(x)$ __Y__ , __J__   13.) $f(-x)$ __X__ , __G__

Inverse: Geometry of graph and algebra of X-change.

Agreement: Geometry of graph and algebra of Y-change.

In the second blanks, write the letter of the matching graph.

f(x)

A.)

B.)

C.)

D.)

E.)

F.)

G.)

H.)

I.)

J.)
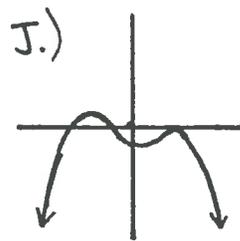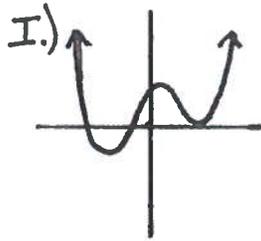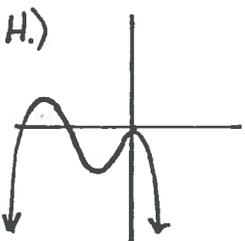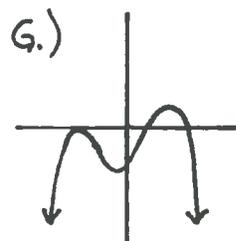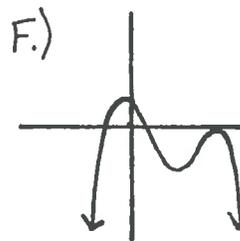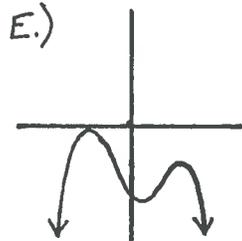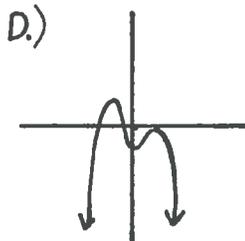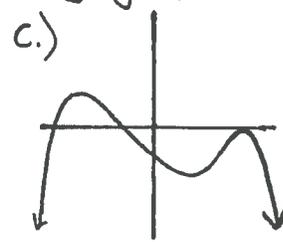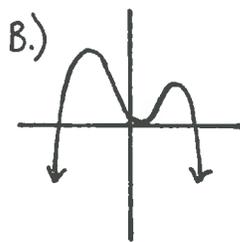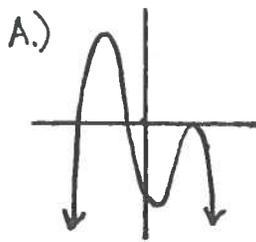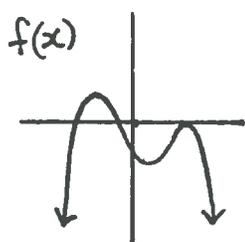
Multiplying Complex Numbers on the Unit Circle
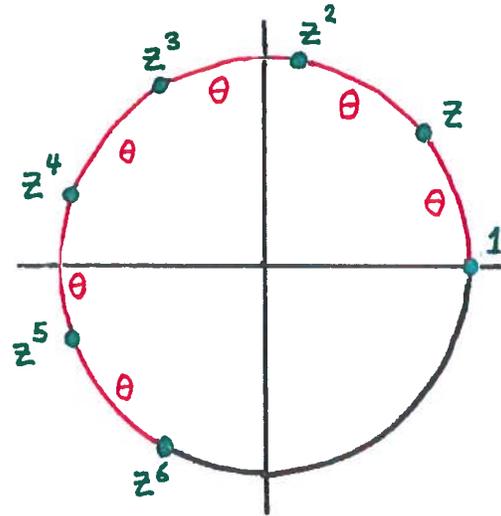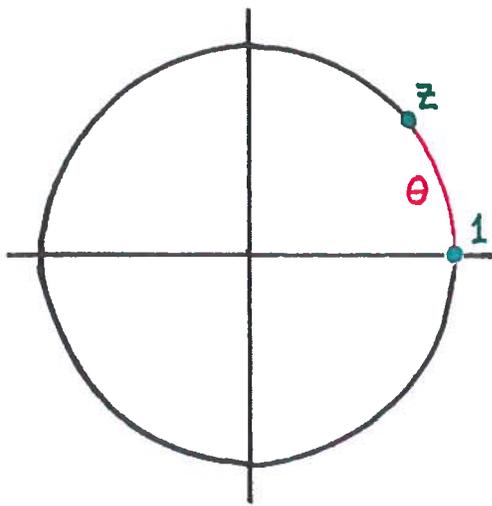
Find the following:

14.) $\left(\dfrac{1}{\sqrt{2}} + i\dfrac{1}{\sqrt{2}}\right)^5 = -\dfrac{1}{\sqrt{2}} - i\dfrac{1}{\sqrt{2}}$

15.) $\left(\dfrac{\sqrt{3}}{2} + i\dfrac{1}{2}\right)^6 = -1$

16.) $\left(-\dfrac{1}{\sqrt{2}} + i\dfrac{1}{\sqrt{2}}\right)^2 = -i$

17.) $\left(\dfrac{1}{2} + i\dfrac{\sqrt{3}}{2}\right)^4 = -\dfrac{1}{2} - i\dfrac{\sqrt{3}}{2}$
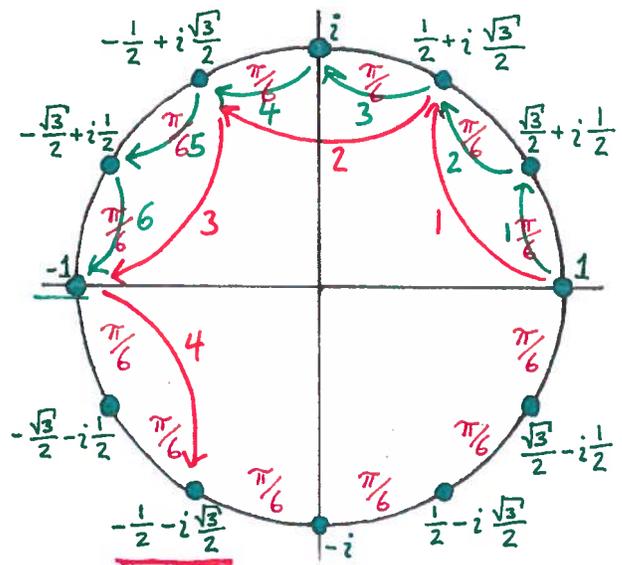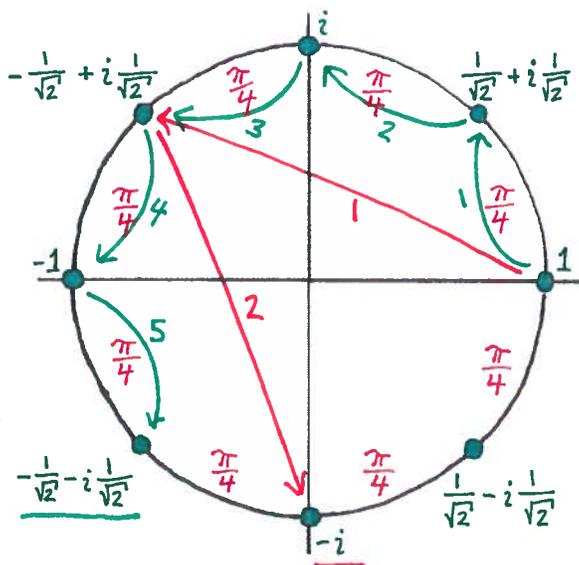
Equations in One Variable

$$h(x)\,f(x) = h(x)\,g(x) \quad\nearrow\quad f(x) = g(x)$$
$$\searrow\quad h(x) = 0$$

For #18-20, write the two equations that must be solved.

18.) $e^{3x}(2x-3) = e^{3x}(4x-7)$ $\nearrow$ $(2x-3) = (4x-7)$
$\searrow$ $e^{3x} = 0$

19.) $(x-4)\log_e(x) = (x-4)5$ $\nearrow$ $\log_e(x) = 5$
$\searrow$ $(x-4) = 0$

20.) $(x^2-5) = (x^2-5)x^3$ $\nearrow$ $1 = x^3$
$\searrow$ $(x^2-5) = 0$

$$a f(x)^2 + b f(x) + c = 0 \quad\longrightarrow\quad f(x) = \frac{-b+\sqrt{b^2-4ac}}{2a}$$
$$\searrow\quad f(x) = \frac{-b-\sqrt{b^2-4ac}}{2a}$$

For #21-22, write the two equations that must be solved.

21.) $2(x-3)^2 - 5(x-3) + 2 = 0$ $\nearrow$ $(x-3) = \dfrac{5+\sqrt{(-5)^2-4(2)(2)}}{2(2)} = 2$
$\searrow$ $(x-3) = \dfrac{5-\sqrt{(-5)^2-4(2)(2)}}{2(2)} = \dfrac{1}{2}$

22.) $\log_e(x)^2 + 3\log_e(x) + 1 = 0$ $\nearrow$ $\log_e(x) = \dfrac{-3+\sqrt{3^2-4(1)(1)}}{2(1)} = \dfrac{-3+\sqrt{5}}{2}$
$\searrow$ $\log_e(x) = \dfrac{-3-\sqrt{3^2-4(1)(1)}}{2(1)} = \dfrac{-3-\sqrt{5}}{2}$